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Title:	Introduction to FEL Simulations
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VUV and X-ray Free-Electron Lasers

Introduction to FEL Simulations

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Wednesday (Jan 27) Lecture Outline

- Canonical formulation of 1D FEL theory
- Q&A
- Numerical simulator for 1D FEL
- Q&A
- Numerical simulator – ZFEL code
- Q&A

Canonical formulation of 1D FEL theory

Lagrangian description

- Lagrangian for a relativistic electron in an electromagnetic field:[1,2]

$$L(\boldsymbol{r},\dot{\boldsymbol{r}};t)=-m_e c^2\sqrt{1-\frac{\dot{\boldsymbol{r}}^2}{c^2}}+e\phi(\boldsymbol{r})-e\dot{\boldsymbol{r}}\cdot\boldsymbol{A}(\boldsymbol{r})$$

- Lagrangian density for classical electrodynamics:[1,3]

$$\mathcal{L}=-\frac{1}{4\mu_0}F^{\alpha\beta}F_{\alpha\beta}-A_\alpha J^\alpha,$$

where, $F^{\alpha\beta}F_{\alpha\beta}=2\left(\partial_\alpha A_\beta\partial^\alpha A^\beta-\partial_\beta A_\alpha\partial^\alpha A^\beta\right)$, $A_\alpha=\left(\frac{\phi(\boldsymbol{r})}{c},\ -\boldsymbol{A}(\boldsymbol{r})\right)$ and $J^\alpha=\left(c\rho(\boldsymbol{r}),\ \boldsymbol{j}(\boldsymbol{r})\right)$;

- The Hamiltonian principle: $\delta S=\delta\int_{t_1}^{t_2}L(\boldsymbol{r},\dot{\boldsymbol{r}};t)dt=0.$

[1] L. D. Landau and E. M. Lifshitz, *The classical theory of fields*; 3rd ed., ser. Course of theoretical physics. Oxford: Pergamon, 1971, Eq. (28.6) [Conversion of Gaussian to SI units is discussed at https://en.wikipedia.org/wiki/Gaussian_units#Major_differences_between_Gaussian_and_SI_units]
[2] https://en.wikipedia.org/wiki/Relativistic_Lagrangian_mechanics and [3] https://en.wikipedia.org/wiki/Covariant_formulation_of_classical_electromagnetism

FEL in Lagrangian description

- An ensemble of electrons creates charge density $\rho(\boldsymbol{r}) = -e\sum_{n=1}^{N_e}\delta\big(\boldsymbol{r} - \boldsymbol{r}_n(t)\big)$ and the current density $j(\boldsymbol{r}) = -e\sum_{n=1}^{N_e}\dot{\boldsymbol{r}}_n(t)\delta\big(\boldsymbol{r} - \boldsymbol{r}_n(t)\big)$ resulting in

$$L_{int} = -\int A_\alpha J^\alpha d^3\boldsymbol{r} = e\sum_{n=1}^{N_e}[\phi(\boldsymbol{r}_n) - \dot{\boldsymbol{r}}_n \cdot \boldsymbol{A}(\boldsymbol{r}_n)],$$

which is the same term as in the Lagrangian for a relativistic electron in an electromagnetic field but summed over all the particles;

- There are no stationary charges therefore $\phi(\boldsymbol{r}) = 0$ will be assumed resulting in Coulomb and Lorentz gauges[1] becoming $\nabla \cdot \boldsymbol{A} = 0$.

[1] https://en.wikipedia.org/wiki/Gauge_fixing

[2] For more info on the Electromagnetic tensor $F^{\alpha\beta}$ go to https://en.wikipedia.org/wiki/Electromagnetic_tensor

Assumptions for 1D FEL

- Consider a helical undulator with a period λ_u that is described by

$$\boldsymbol{A}_u(z) = \frac{m_e c}{\sqrt{2}e} K \hat{\boldsymbol{\epsilon}} e^{-ik_u z} + c.c.,$$

where $\hat{\boldsymbol{\epsilon}} = (\hat{\boldsymbol{x}} + i\hat{\boldsymbol{y}})/\sqrt{2}$ is the polarization vector, $k_u = 2\pi/\lambda_u$ is the undulator wavenumber and $K = 0.934\ B_0[T]\lambda_u[cm]$ is the undulator parameter;

- Further consider a plane-wave radiation field of wavelength λ:

$$\boldsymbol{A}_r(z,t) = -\frac{i}{\sqrt{2}k} E(t) \hat{\boldsymbol{\epsilon}} e^{ikz - i\omega t} + c.c.,$$

where the phase conversion is such that $\boldsymbol{A}_u \cdot \boldsymbol{A}_r \neq 0$.

- Since the Lagrangian does not depend on x or y variables then the canonical momenta $\frac{\partial L}{\partial \dot{\boldsymbol{r}}_\perp} \stackrel{\text{def}}{=} \boldsymbol{p}_\perp = const$ and could be set to zero.

1D FEL in Lagrangian description

- The undulator vector potential is externally created and does not have to be included in the Lagrangian of the system. The Lagrangian of the radiation field confined to some volume V is

$$L = \int \mathcal{L} d\boldsymbol{r} = -\frac{1}{2\mu_0}\left(B_r^2 - \frac{E_r^2}{c^2}\right)V \approx \frac{V}{\mu_0}\frac{iE^*}{\omega}\dot{E},$$

which results in the canonical momentum for the electromagnetic field to be

$$p_E \stackrel{\text{def}}{=} \frac{dL}{d\dot{E}} = \frac{V}{\mu_0}\frac{iE^*}{\omega};$$

- The Hamiltonian principle can now be rewritten as

$$\delta\left(\int_{t_1}^{t_2} p_E dE + \sum_{n=1}^{N_e} p_{z,n} dz_n - H_n dt\right) = 0.$$

1D FEL in Hamiltonian description

- The Hamiltonian for a single electron becomes

$$H_n \approx c\sqrt{m^2c^2 + p_{z,n}^2 + 2\frac{e\,m_e K}{k} Im[E(t)e^{i\theta_n}]},$$

where $\theta_n = (k + k_u)z_n - \omega t$ is the ponderomotive phase; $m^2 = m_e^2(1 + K^2)$ is a longitudinal mass of an 'undulator' electron and we have neglected $\sim |E(t)|^2$ term;

- We will replace the canonical variable z_n with θ_n in the Hamiltonian principle such that $dz_n = (d\theta_n + \omega dt)/(k + k_u)$;
- We will define the efficiency of an FEL interaction $\rho = \frac{E_{sat}^2 V}{\mu_0}/\gamma_0 m_e c^2 N_e$ as the ratio of the EM energy at saturation to the beam energy and rescale the field amplitude to its value at saturation $dE = E_{sat} dA$.

1D FEL in Hamiltonian description cont'd

- The Hamiltonian principle in the FEL theory notations becomes

$$\delta\left(\int_{\tau_1}^{\tau_2} p_A dA + \sum_{n=1}^{N_e} p_{\theta,n} d\theta_n - \widetilde{H}_n d\tau\right) = 0$$

in terms of scaled time $dt = {}^{d\tau}/_{2\rho k_u c}$, new canonical momenta $p_{\theta,n} = \frac{p_{z,n}}{k+k_u}$ and $p_A = i\frac{\rho\gamma_0 m_e c}{k} N_e A^*$, and a new Hamiltonian for n^th electron

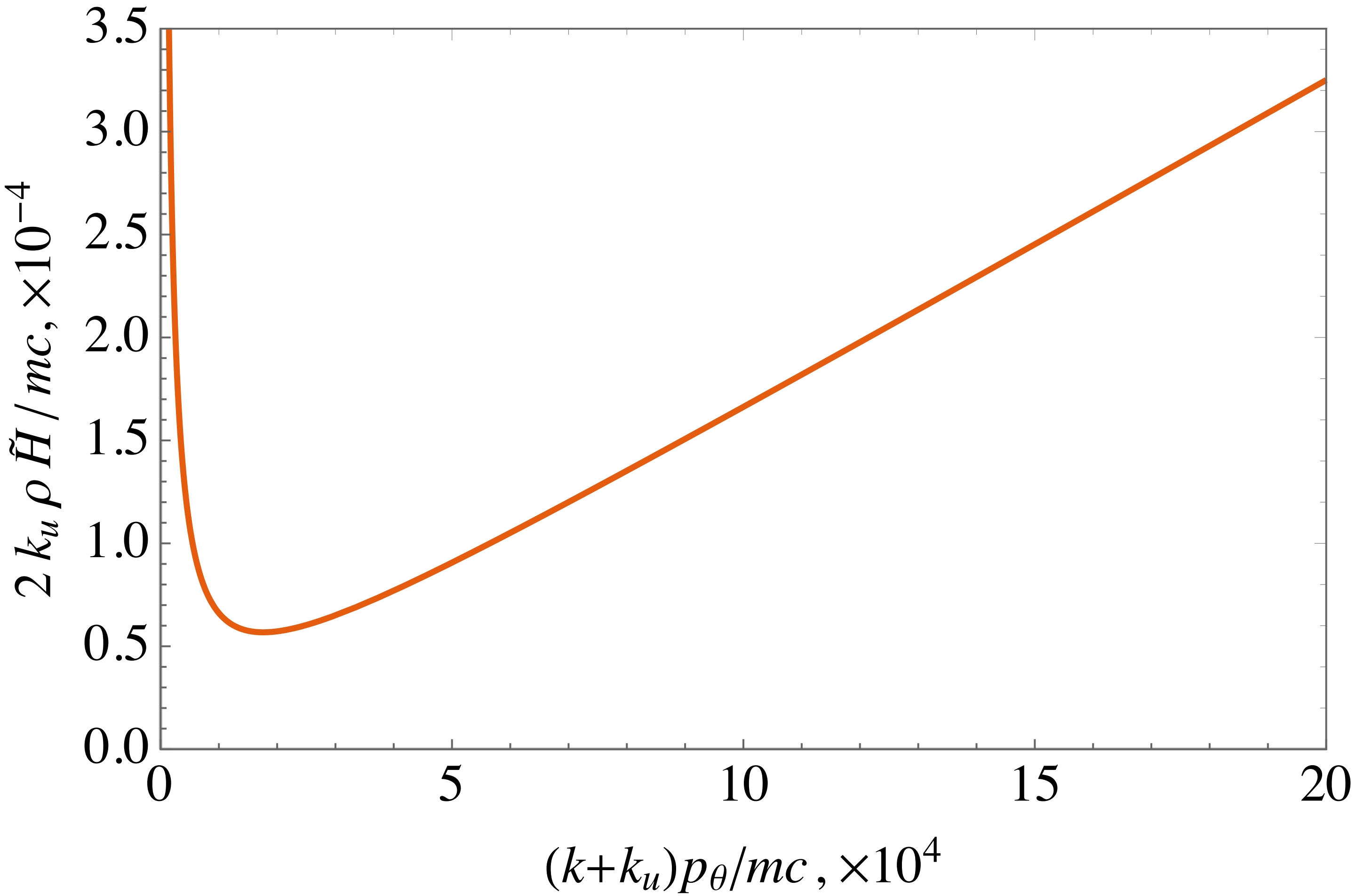
$$\widetilde{H}_n = \frac{1}{2k_u\rho}\left(\sqrt{(k+k_u)^2 p_{\theta,n}^2 + m^2c^2(1+\mathcal{V})} - k\, p_{\theta,n}\right),$$

where a scaled ponderomotive potential has been introduced

$$\mathcal{V} = \frac{2K}{1+K^2}\frac{\omega_{pe}}{\omega}\sqrt{\rho\gamma_0} Im\left[A(\tau)e^{i\theta_n}\right],$$

in terms of the beam plasma frequency $\omega_{pe}^2 = \frac{e^2}{m_e\varepsilon_0}\frac{N}{V}$.

FEL Hamiltonian



- In the absence of the ponderomotive potential created by the radiation the Hamiltonian of n[th] electron becomes:

$$\widetilde{H} = \frac{mc}{2k_u\rho}\left(\sqrt{1+p^2} - v_{BR}p\right),$$

where a new momentum $p = \frac{(k+k_u)p_\theta}{mc}$ and the Bambini-Renieri velocity in units of speed of light $v_{BR} = \frac{k}{k+k_u}$ have been introduced;

- We here illustrate the case of $\lambda = 0.3\ A$ radiation in $\lambda_u = 1.86\ cm$ undulator.

Ponderomotive Potential

- The scaled ponderomotive potential $\mathcal{V} = \frac{2K}{1+K^2}\frac{\omega_{pe}}{\omega}\sqrt{\rho\gamma_0}Im\left[A(\tau)e^{i\theta_n}\right] \ll 1$ can be treated as a perturbation to the Hamiltonian evolution:

$$\frac{dp_\theta}{d\tau} = -\frac{\partial\widetilde{H}}{\partial\theta} = 0$$

$$\frac{d\theta}{d\tau} = \frac{\partial\widetilde{H}}{\partial p_\theta} = \frac{k+k_u}{2k_u\rho}\left(\frac{p}{\sqrt{1+p^2}} - v_{BR}\right)$$

where the canonical momentum is conserved and the phase is linearly increasing;

- Efficient interaction with the Ponderomotive potential requires $\langle\mathcal{V}\rangle_\theta \neq 0$, which could be achieved with $\frac{d\theta}{d\tau} = 0 = \frac{\partial\widetilde{H}}{\partial p_\theta}$ condition that corresponds to the minimum of $\widetilde{H}$!

The resonant energy and the FEL parameter

- The equilibrium momentum that minimizes the Hamiltonian and maximizes the ponderomotive interaction can be expressed as $p_\theta^{eq} = \frac{\gamma_{eq} m \dot{z}_{eq}}{k+k_u}$ with an equilibrium Lorentz factor γ_{eq} corresponding to the velocity $\dot{z}_{eq} = c\, v_{BR}$;
- This implies that the resonant energy for an electron is $\gamma_r^2 \approx (1+K^2)k/2k_u$ in accordance with the classical X-ray FEL theory!
- The final Hamiltonian thus becomes

$$H_n = H_0 + \frac{p_n^2}{2M} + 2M\, Im\left[A(\tau)e^{i\theta_n}\right],$$

where $H_0 = mc^2/2k_u c\rho\gamma_{eq}$, p_n is the detuning for the equilibrium momentum, and $M = \rho p_\theta^{eq}$ if one chooses the FEL parameter to be $\rho = \frac{1}{\gamma_r}\left(\frac{K\omega_{pe}}{4ck_u}\right)^{\frac{2}{3}}$!

1D FEL equations

$$\frac{dA}{d\tau} = \frac{\partial}{\partial p_A}\sum_{n=1}^{N_e} H_n \approx \frac{1}{N_e}\sum_{n=1}^{N_e} e^{-i\theta_n}$$
$$\frac{d\theta_n}{d\tau} = \frac{\partial H_n}{\partial p_n} = \frac{p_n}{M}$$
$$\frac{dp_n}{d\tau} = -\frac{\partial H_n}{\partial \theta_n} = -2M\, Re\left[A(\tau)e^{i\theta_n}\right]$$

One can introduce a scaled energy detuning $\eta_n = \frac{p_n}{M} = \frac{\gamma_n - \gamma_r}{\rho\gamma_r}$ in order to recover the Eqs. 4.31a, 4.31b, 4.31c and 4.31d of the book.

Additional reading on The FEL Hamiltonian

- S. D. Webb, “Period-Averaged Symplectic Maps for the FEL Hamiltonian” 38th International Free Electron Conference, 2017;
- P. M. Anisimov, “Canonical Formulation of 1D FEL Theory Revisited, Quantized and Applied to Electron Evolution”, 38th International Free Electron Conference, 2017;
- P. M. Anisimov, “Quantum theory for 1D X-ray Free Electron Laser”, Journal of Modern Optics, 65(11), pp 1370-1377, 2018.

Numerical simulator for 1D FEL

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1D FEL equations in Python

$$\frac{dA}{d\tau} = \frac{1}{N_e}\sum_{n=1}^{N_e} e^{-i\theta_n}$$

$$\frac{d\theta_n}{d\tau} = \eta_n$$

$$\frac{d\eta_n}{d\tau} = -2\, Re\left[A(\tau)e^{i\theta_n}\right]$$

```python
from scipy.integrate import solve_ivp
import numpy as np
import matplotlib.pyplot as plt

def rhs(t, y):
    """
    The right-hand side of the 1D canonical FEL equations;
    t - the current time;
    y - array of [A, theta, eta]
    """
    n = len(y)//2
    A = y[0]
    theta = y[1:n+1]
    eta = y[n+1:]
    dA_dt = np.mean(np.exp(-1j*theta))
    dtheta_dt = eta
    deta_dt = -2*np.real(A*np.exp(1j*theta))
    return np.concatenate(([dA_dt],
                           dtheta_dt,
                           deta_dt))

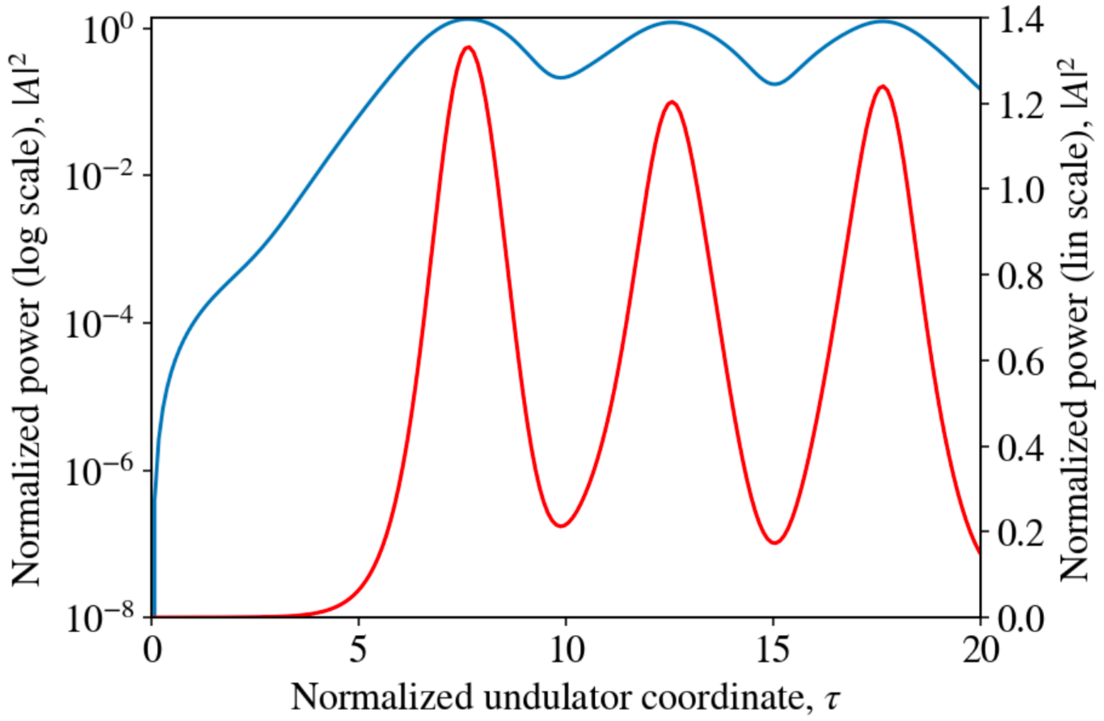
sol = solve_ivp(rhs, [0, 4*np.pi],
                np.concatenate(([A0+0j], theta0, eta0)),
                max_step=0.1)
```

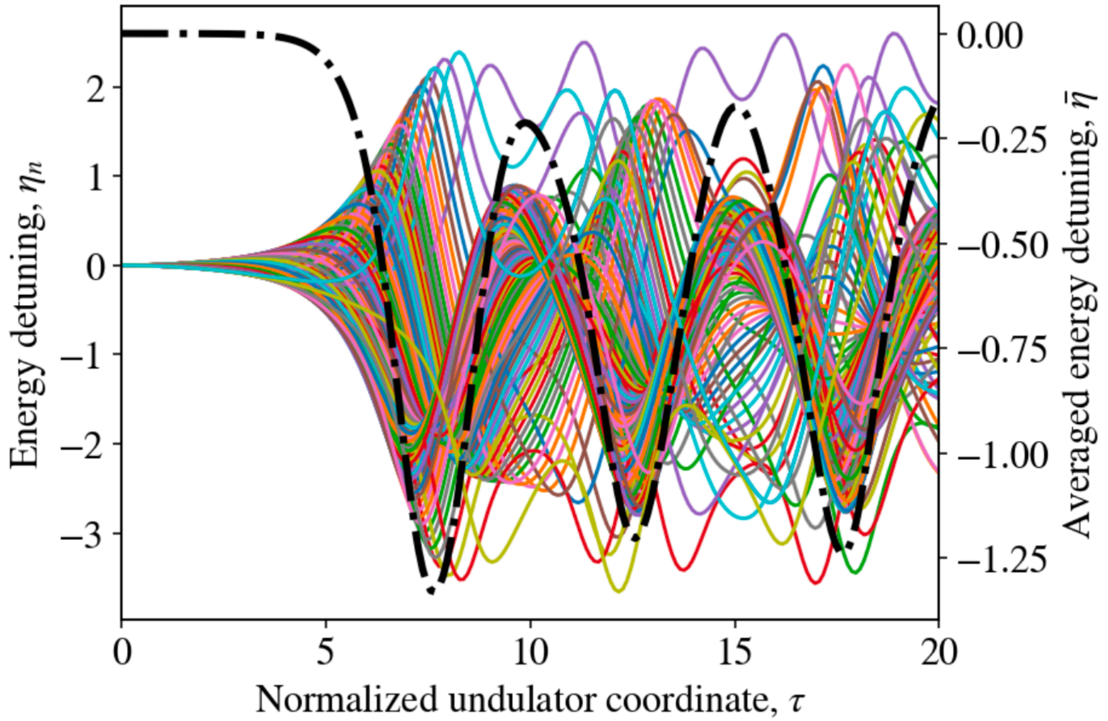
What are the initial conditions?

- $A(0) = A_0$ corresponds to a seeded FEL and $A(0) = 0$ corresponds to SASE FEL;
- $\theta_n = 0$ gives bunching $b = \frac{1}{N_e}\sum_{n=1}^{N_e} e^{-i\theta_n} = 1$;
- A random uniform distribution $\theta_n \in [-\pi, \pi)$ has bunching $b = \frac{1}{N_e}\sum_{n=1}^{N_e} e^{-i\theta_n} = 0$ but $\langle |b|^2 \rangle_\theta = 1/N_e$, which is sufficient to initiate FEL instability;
- $\theta_n = x_n + \delta\theta \sin(x_n)$ such that $\langle |b|^2 \rangle = 0$ when $\delta\theta = 0$ – a so called 'quiet start'.
- What other initial conditions can you think of?

Case 1

```
ne = 100
tau = 20
A0 = 0
theta0 = np.linspace(-4*np.pi, 4*np.pi, ne)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
plot(sol)
```





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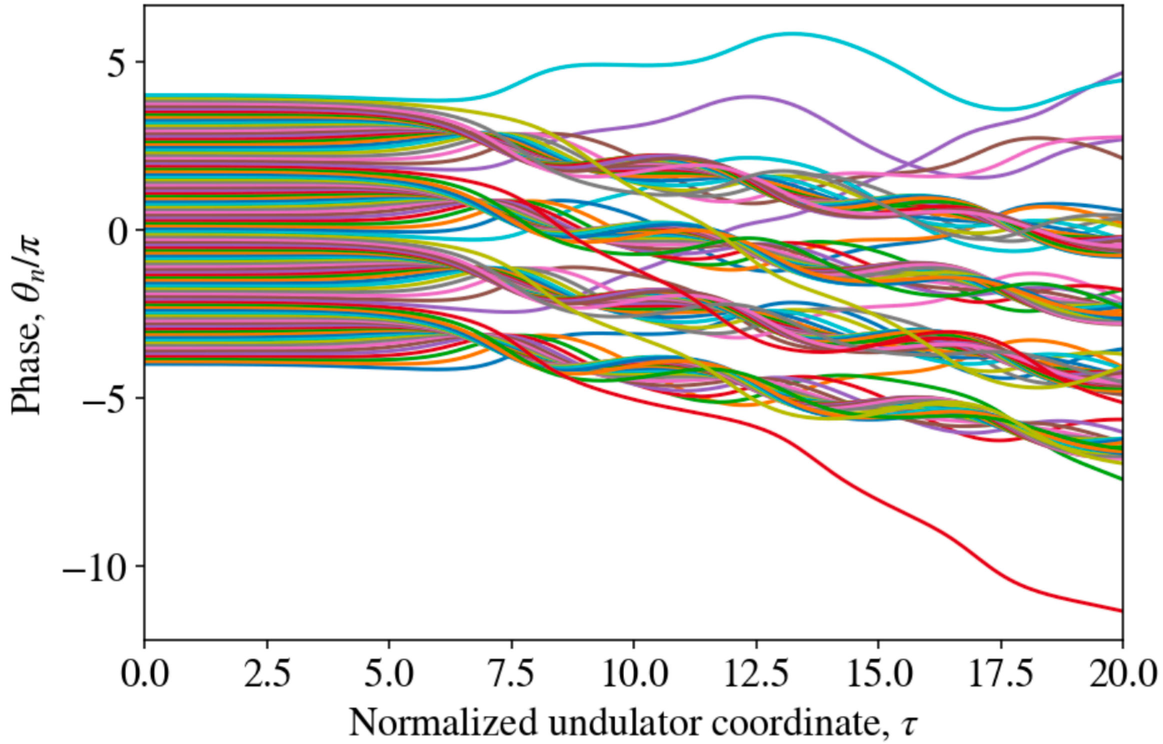
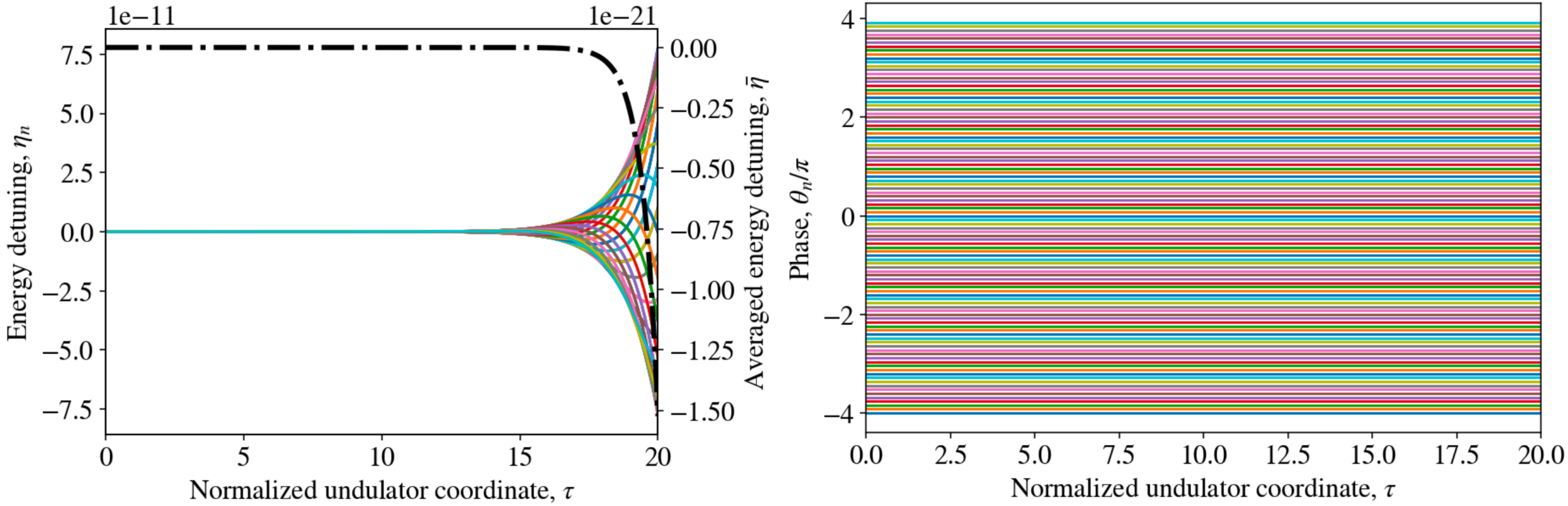


Case 2 – “quiet start”

```
ne = 100
tau = 20
A0 = 0
theta0 = np.linspace(-4*np.pi, 4*np.pi, ne, endpoint=False)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
plot(sol)
```



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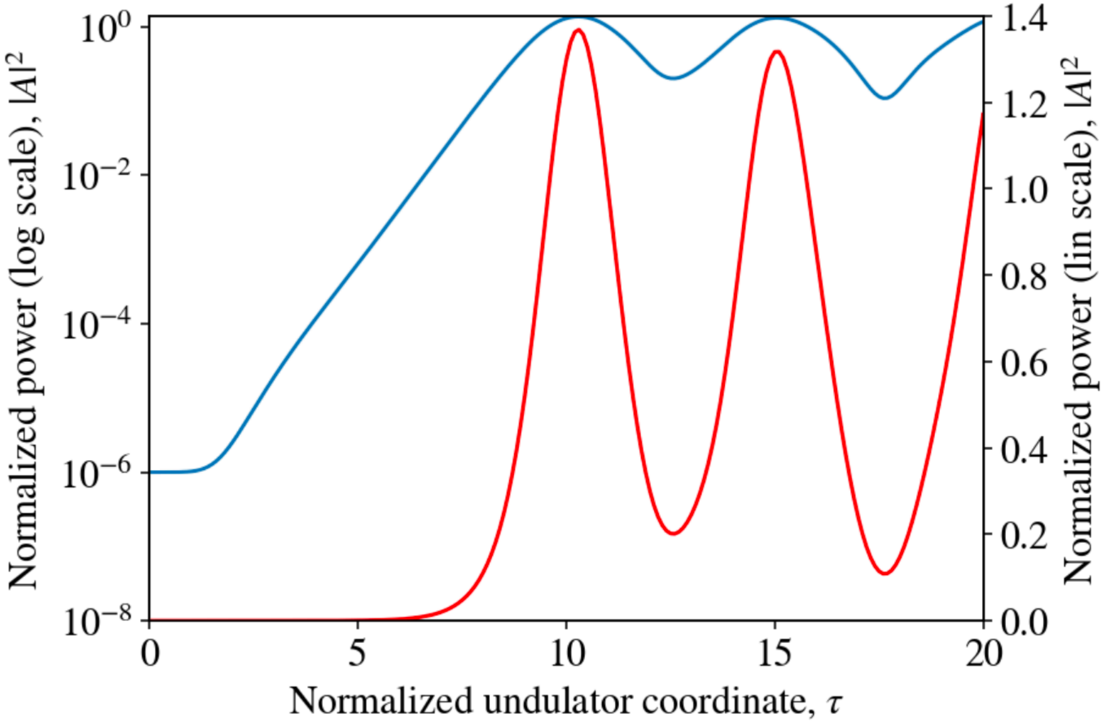
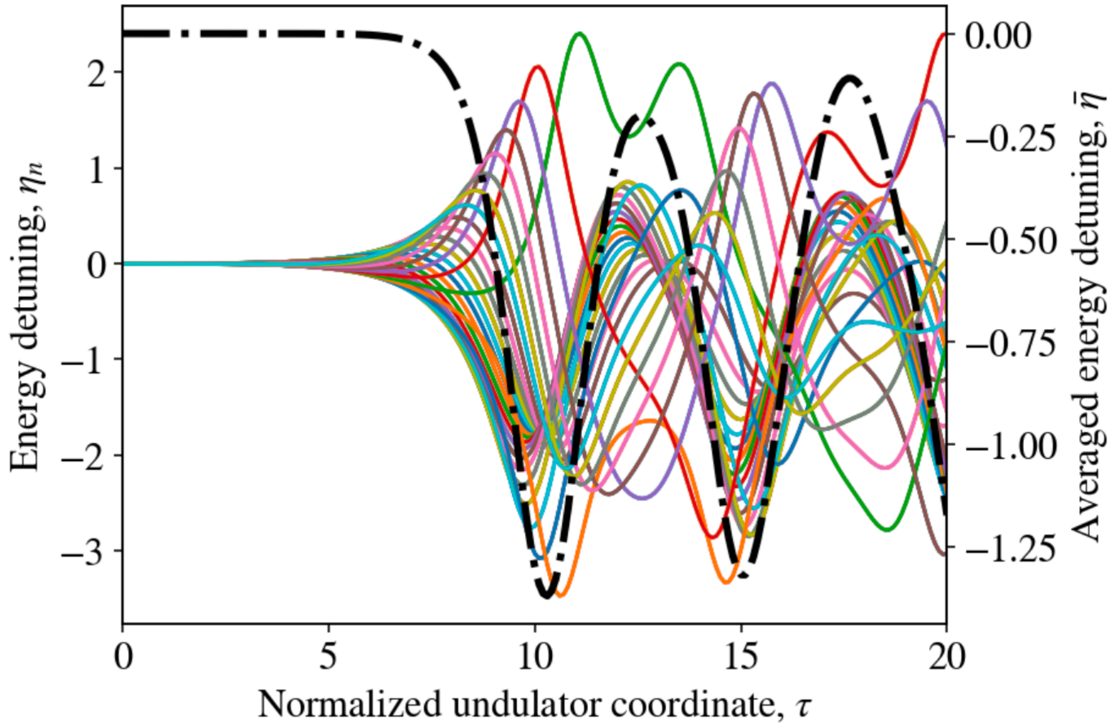
Case 3 – "seeded operation"

```
ne = 100
tau = 20
A0 = 1e-3
theta0 = np.linspace(-4*np.pi, 4*np.pi, ne, endpoint=False)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
plot(sol)
```





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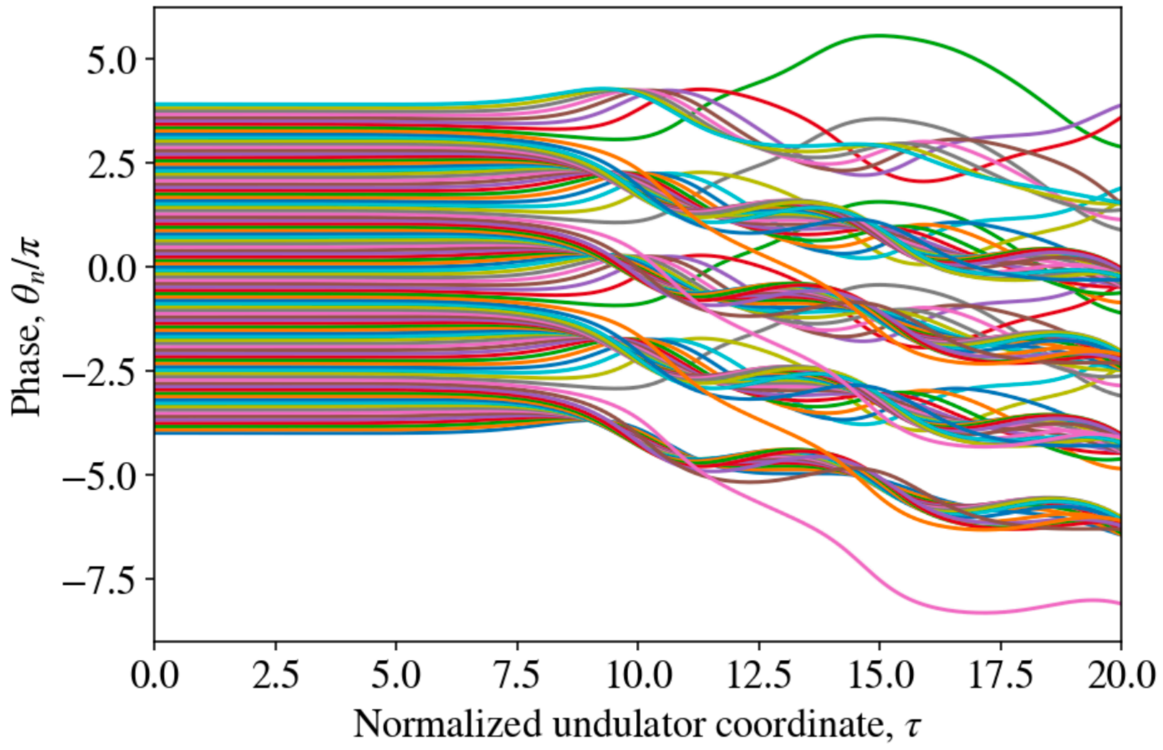
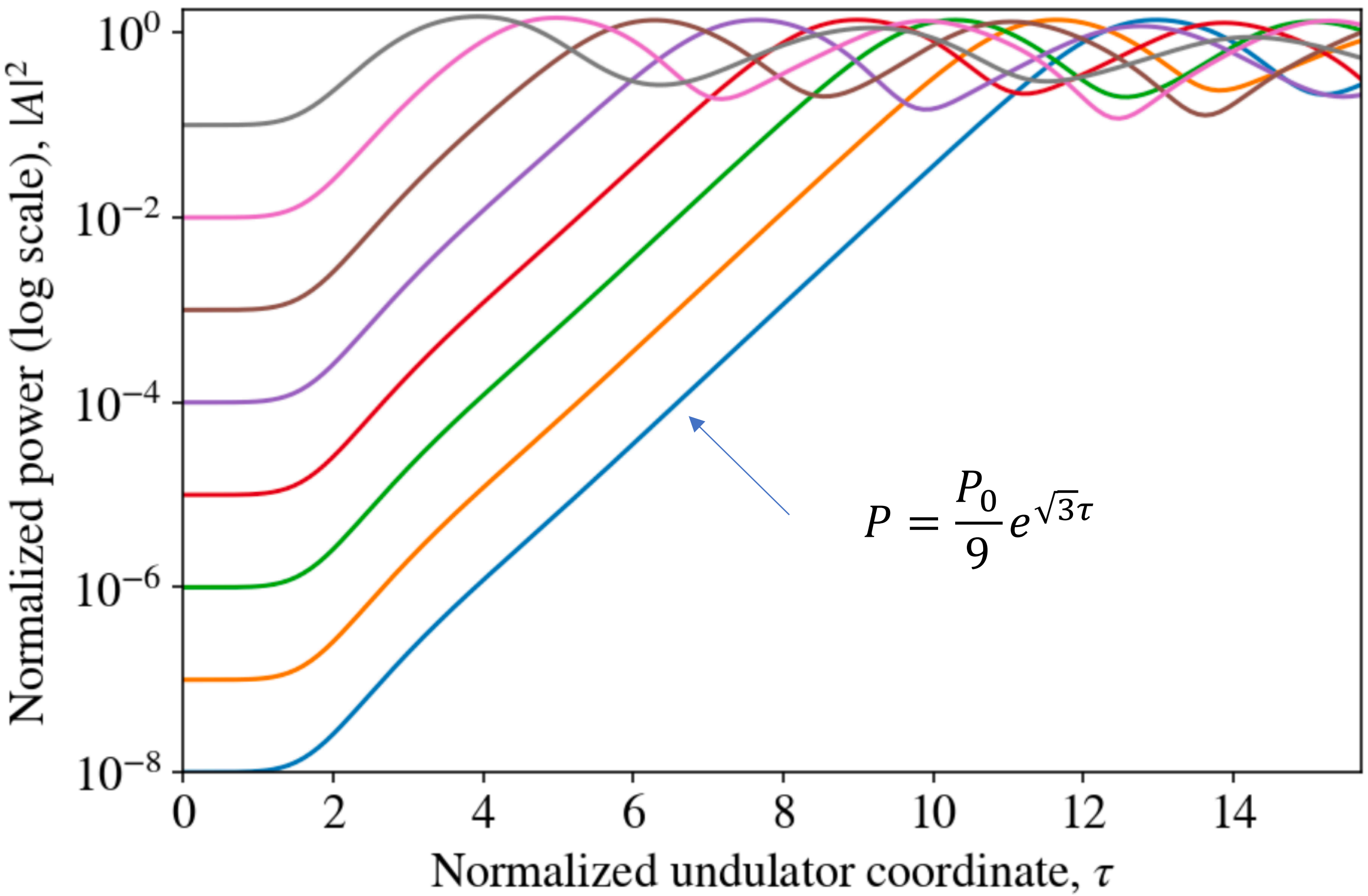
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
plot(sol)
```



Case 3 – “seeded operation” scan



Observe:

- Lethargy period;
- Exponential growth;
- Saturated power;
- Power circulation.

Compare:

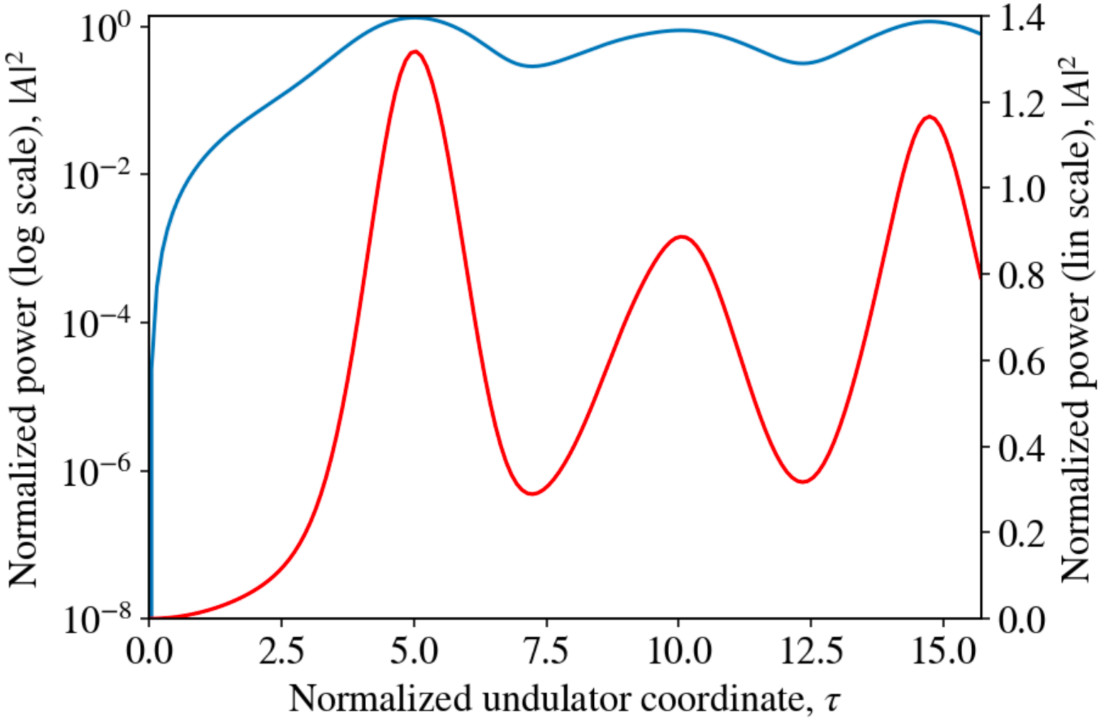
$$P = \frac{P_0}{9} \left| e^{\frac{i+\sqrt{3}}{2}\tau} + e^{\frac{i-\sqrt{3}}{2}\tau} + e^{-i\tau} \right|^2$$

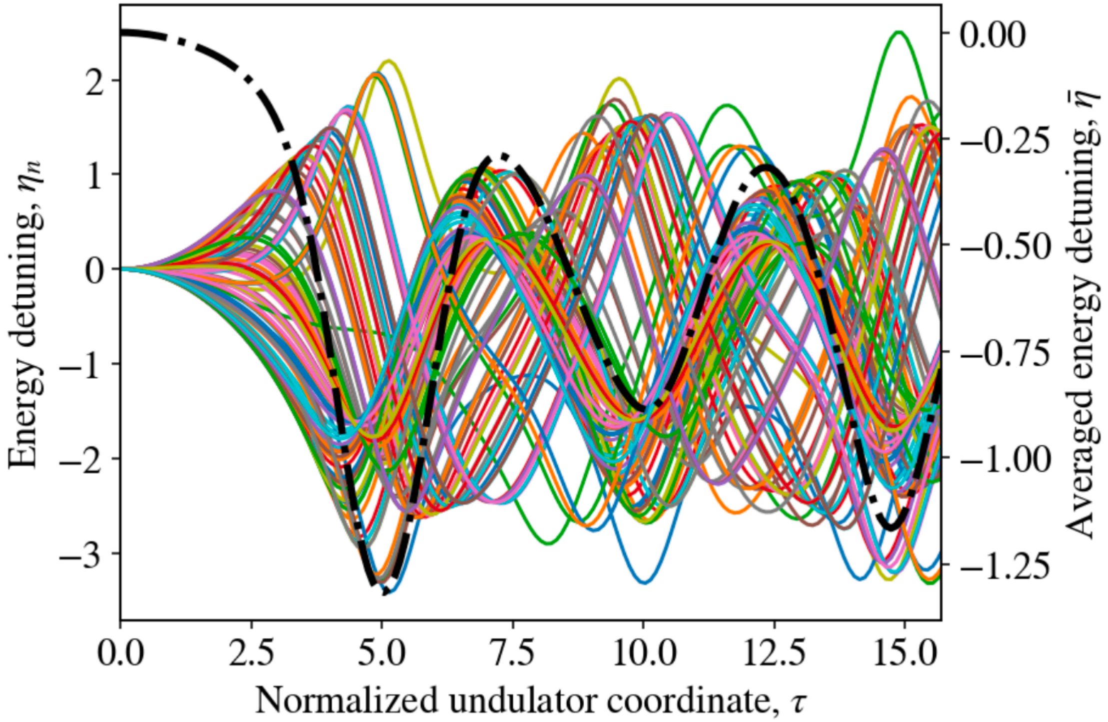
Invariant of the evolution:

$$\eta(\tau) + |A(\tau)|^2 = const$$

Case 4 – "SASE operation"

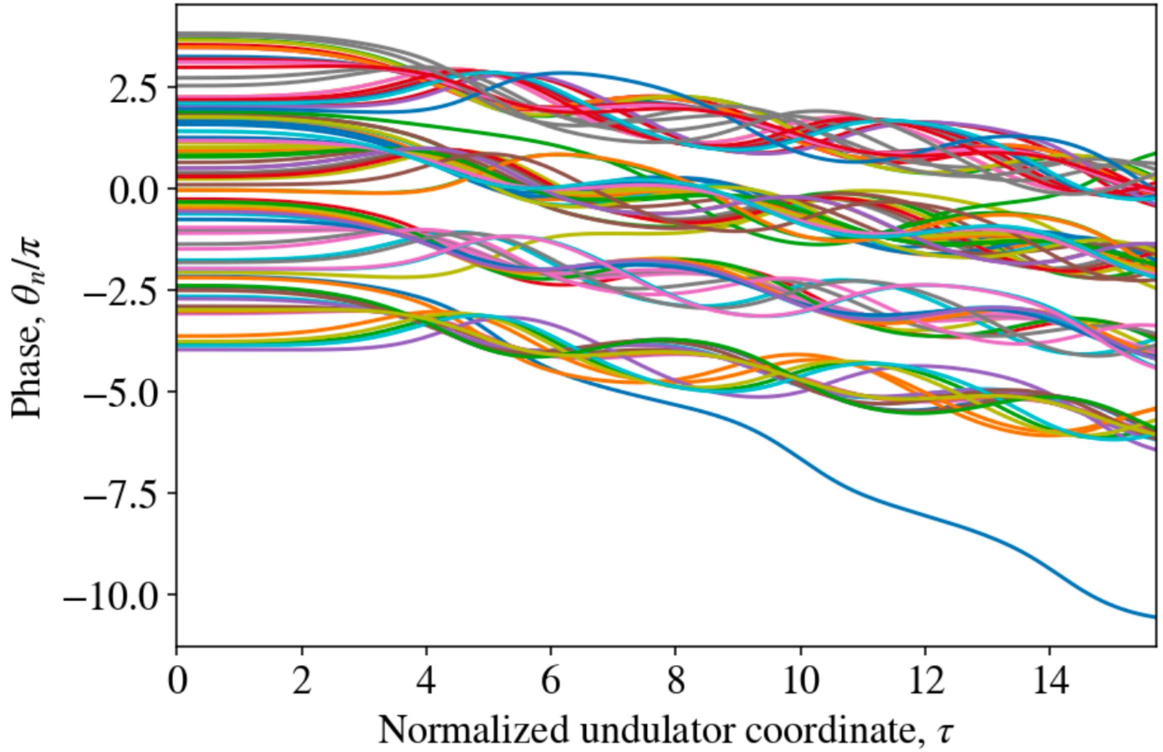
```
ne = 100
tau = 5*np.pi
A0 = 0
theta0 = np.random.uniform(low=-4*np.pi, high=4*np.pi, size=ne)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
plot(sol)
```





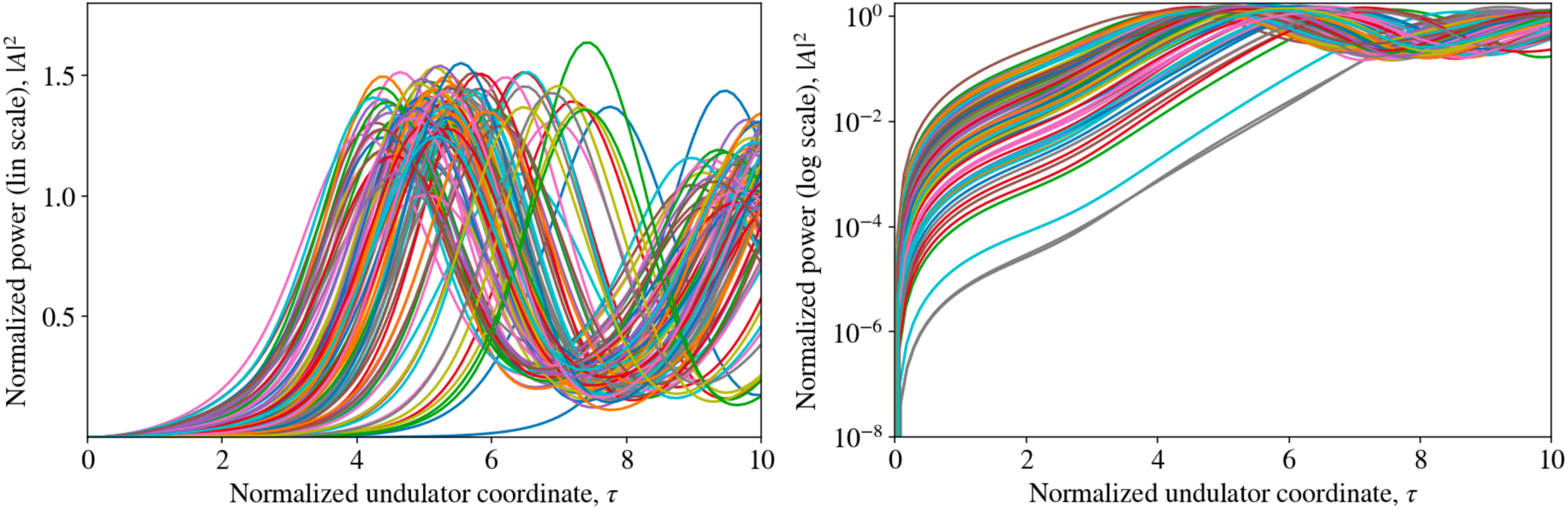
Case 4 – "SASE operation"

```
ne = 100
tau = 5*np.pi
A0 = 0
theta0 = np.random.uniform(low=-4*np.pi, high=4*np.pi, size=ne)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
plot(sol)
```



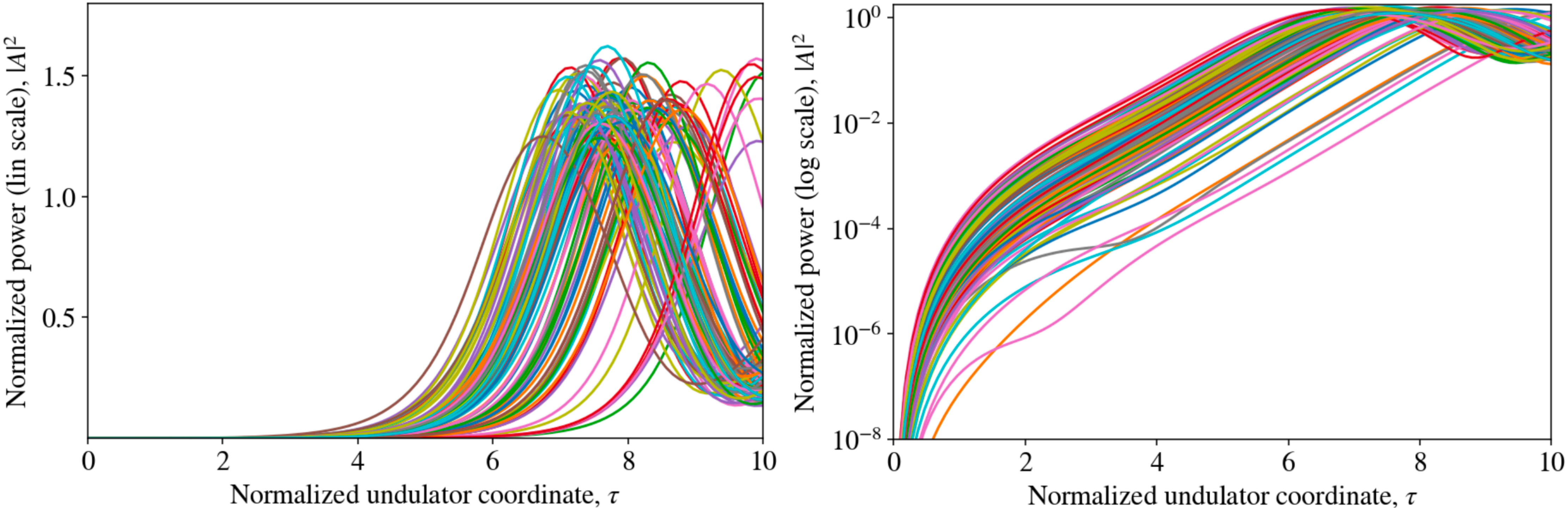
Case 4 – "SASE operation" scan

```
ne = 100
tau = 5*np.pi
A0 = 0
theta0 = np.random.uniform(low=-4*np.pi, high=4*np.pi, size=ne)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
plot(sol)
```



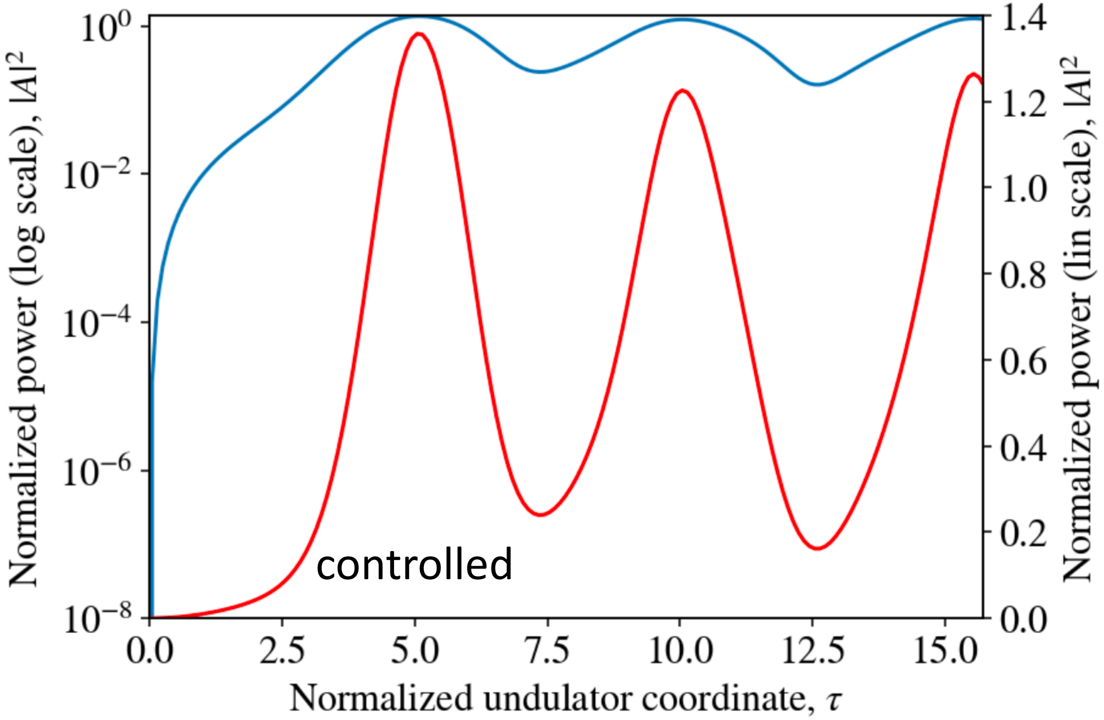
Case 5 – “Energy spread driven FEL” scan

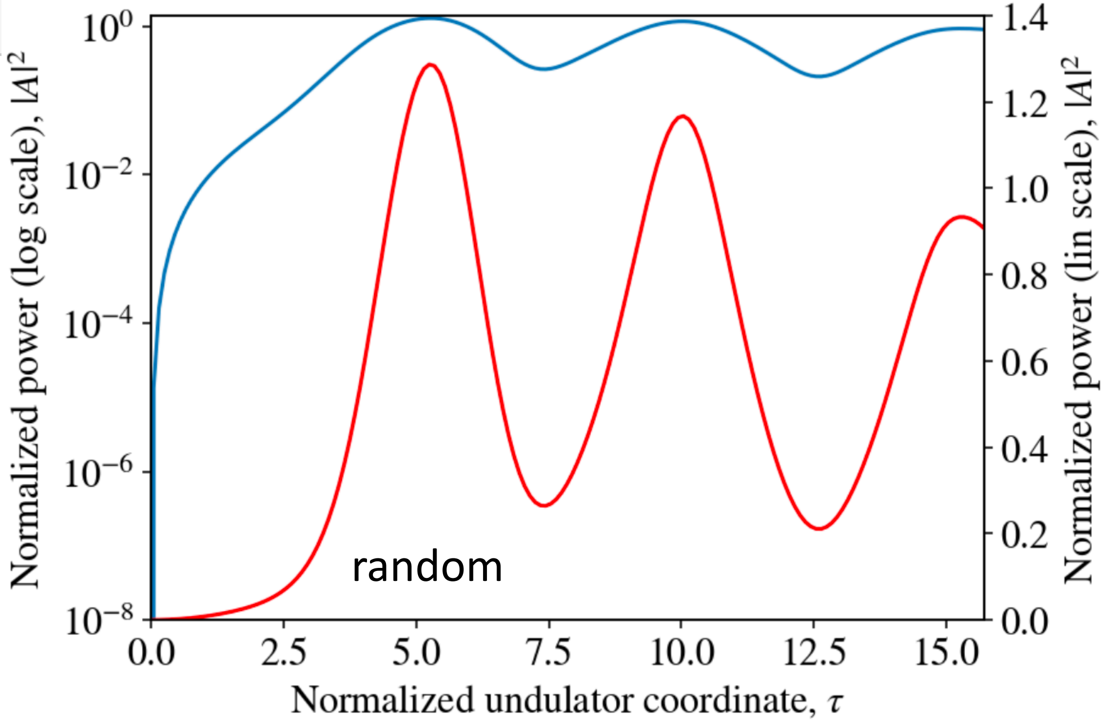
```
ne = 100
tau = 5*np.pi
A0 = 0
theta0 = np.linspace(-4*np.pi, 4*np.pi, ne, endpoint=False)
p0 = np.random.normal(0, 0.1, ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
plot(sol)
```



Case 6 – "controlled SASE"

```
ne = 100
tau = 5*np.pi
A0 = 0
dtheta = 2/np.sqrt(100)
theta0 = np.linspace(-4*np.pi, 4*np.pi, ne, endpoint=False)
theta0 -= dtheta*np.sin(theta0)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
```





Case 6 – “controlled SASE”

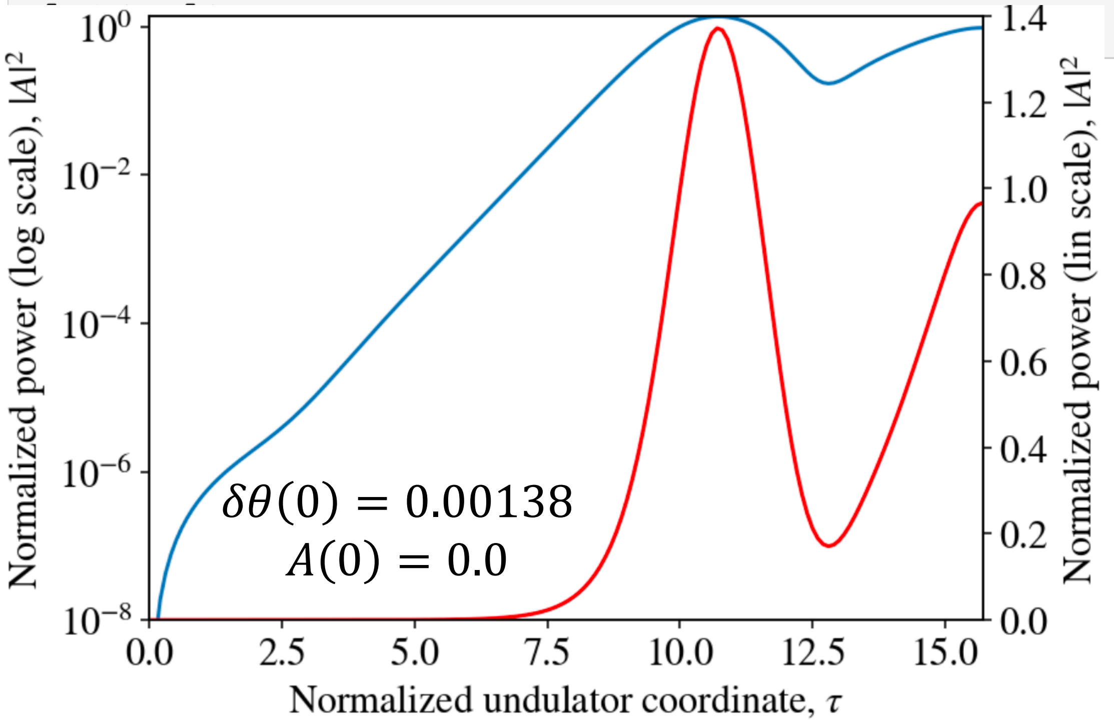
```
ne = 100
tau = 5*np.pi
A0 = 0
dtheta = 2/np.sqrt(100)
theta0 = np.linspace(-4*np.pi, 4*np.pi, ne, endpoint=False)
theta0 -= dtheta*np.sin(theta0)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
plot(sol)
```

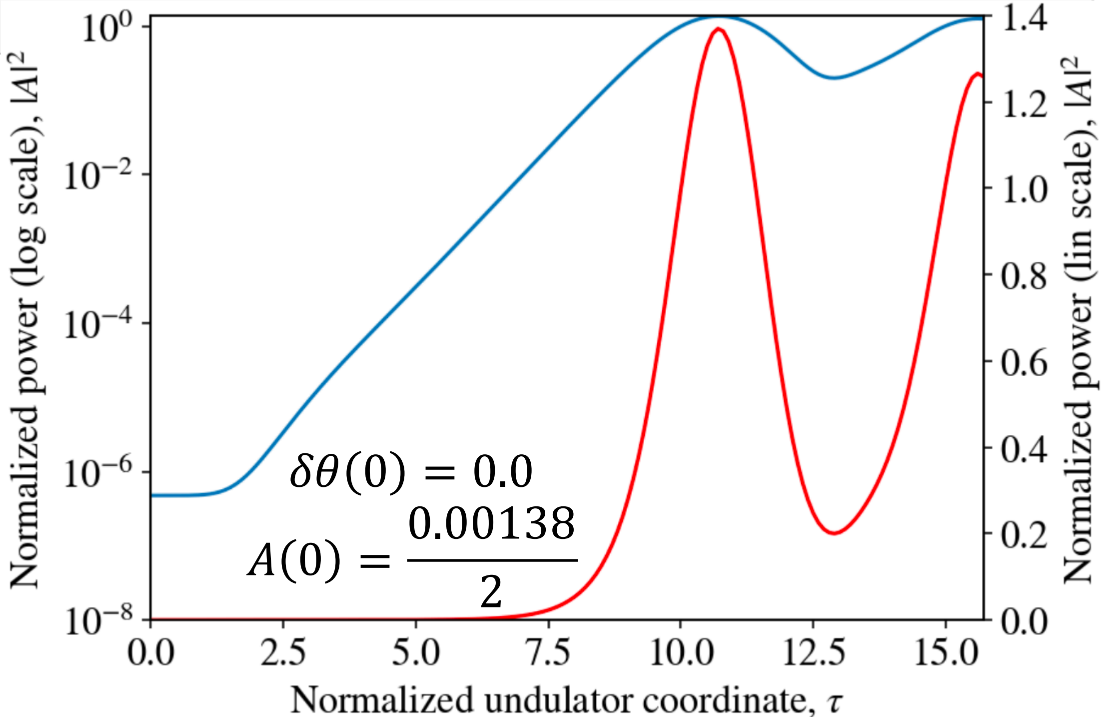
- The choice of $\delta\theta = 2/\sqrt{N_e}$ comes from the requirement to recover the correct Poisson statistics[1] such that $\left|\langle e^{-i\theta_n}\rangle\right| = \frac{1}{2}\delta\theta = 1/\sqrt{N_e}$;
- N_e is the number of interacting electrons such that $N_e = 4.3\,\frac{L_g}{\lambda_u}\frac{I_b\lambda}{ev_b}$ and $\delta\theta = 1.38{\times}10^{-3}$ corresponds to MaRIE XFEL case;
- We can also define an equivalent startup noise power $|A(0)|^2 = N_e^{-1}$.

[1]We refer an interested reader to “The effect of shot noise on the start up of the fundamental and harmonics in free-electron lasers” by H. P. Freund et al., J. Appl. Phys. 104, 123114 (2008); https://doi.org/10.1063/1.3040689

Case 7 – SASE vs shot noise power

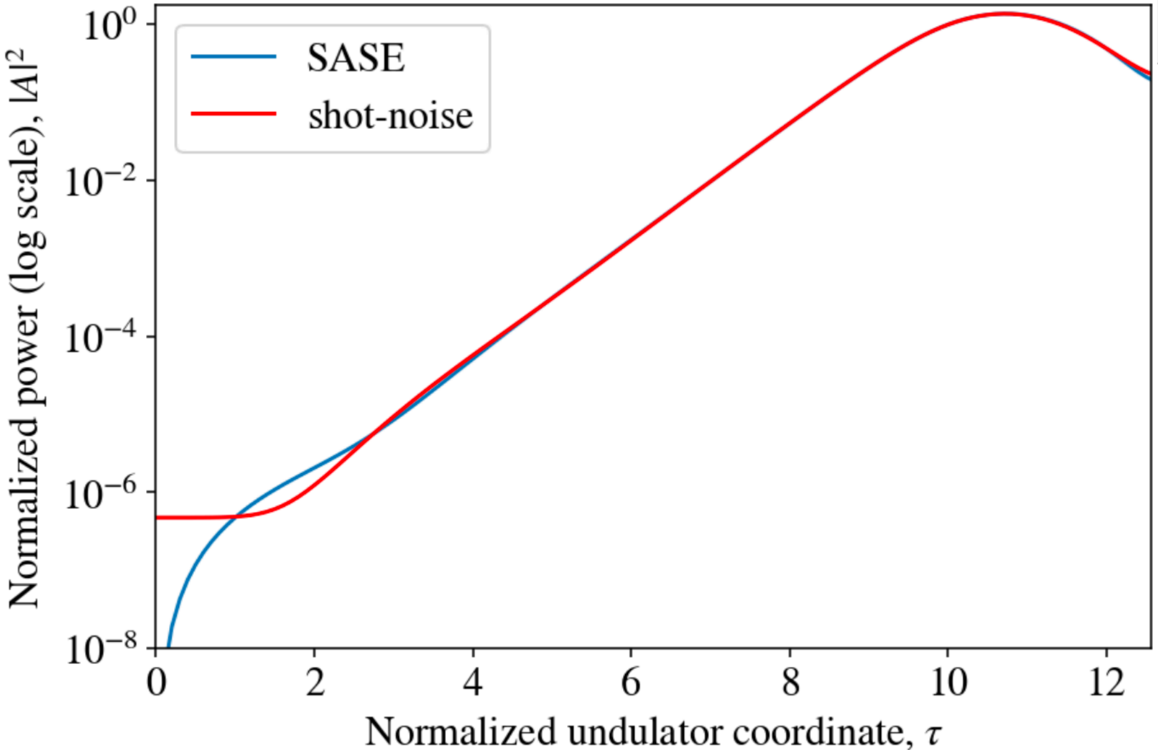
```
ne = 25
tau = 5*np.pi
A0 = 0
dtheta = 1.38e-3
theta0 = np.linspace(-4*np.pi, 4*np.pi, ne, endpoint=False)
theta0 -= dtheta*np.sin(theta0)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
```





Case 7 – SASE vs shot noise power

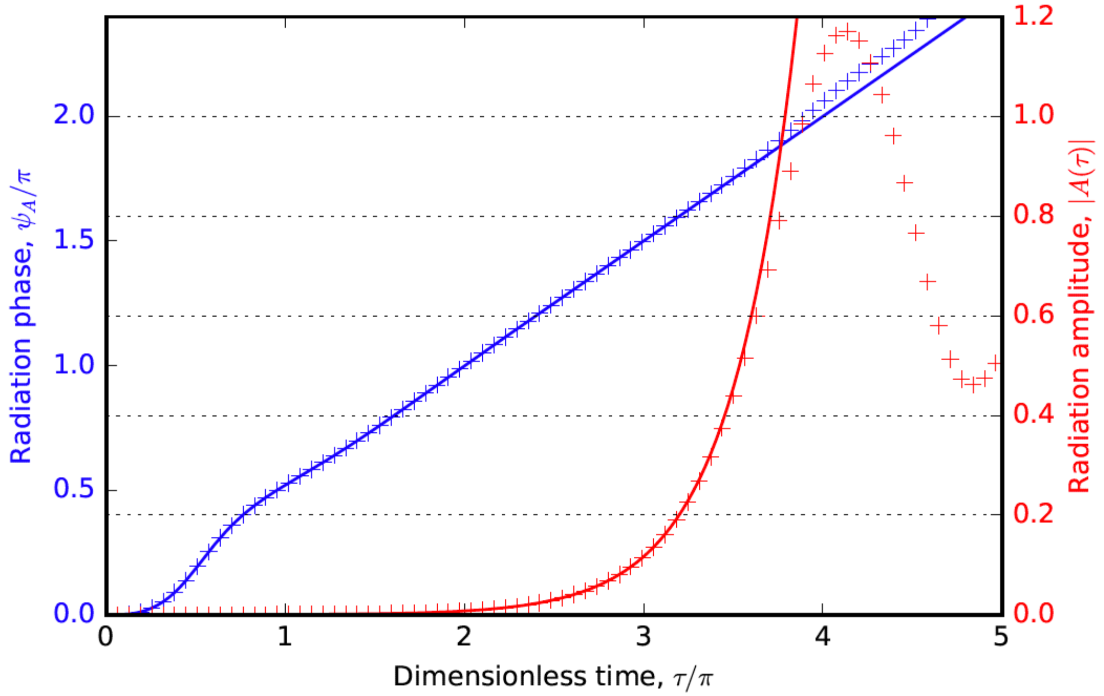
```
ne = 25
tau = 5*np.pi
A0 = 0
dtheta = 1.38e-3
theta0 = np.linspace(-4*np.pi, 4*np.pi, ne, endpoint=False)
theta0 -= dtheta*np.sin(theta0)
p0 = np.zeros(ne)
sol = solve_ivp(rhs, [0, tau], np.concatenate(([A0+0j], theta0, p0)), max_step=0.1)
```



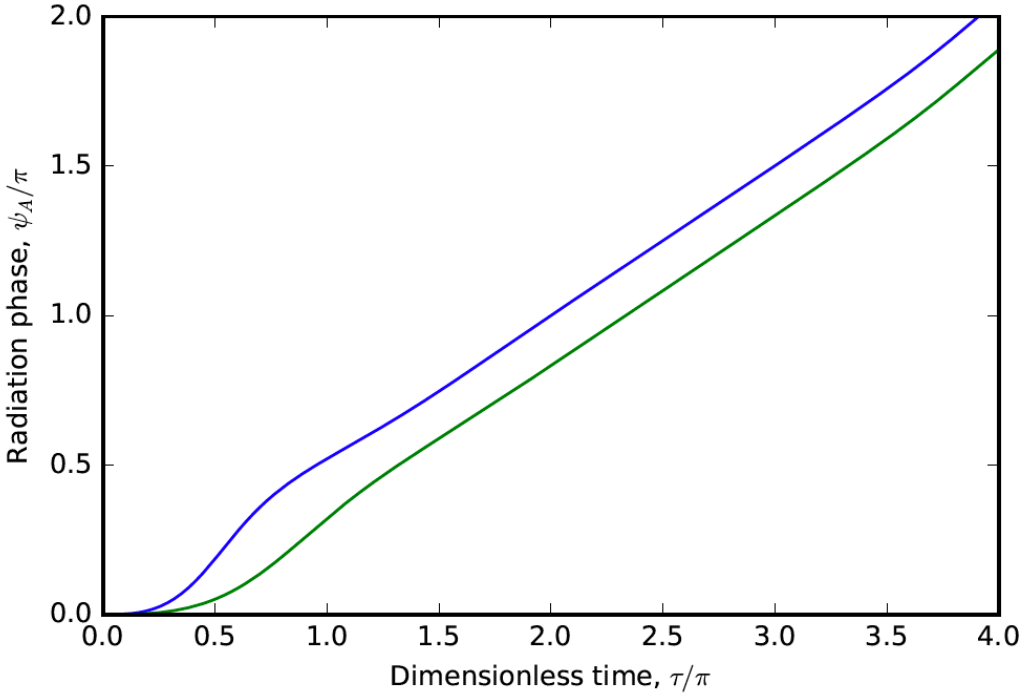
Observe:

- SASE vs shot noise power distinction happens during the lethargy period;
- The number of macro particles used for the simulation does not depend on the number of interacting electrons $n_e \ll N_e$!

Case 7 – SASE vs shot noise power

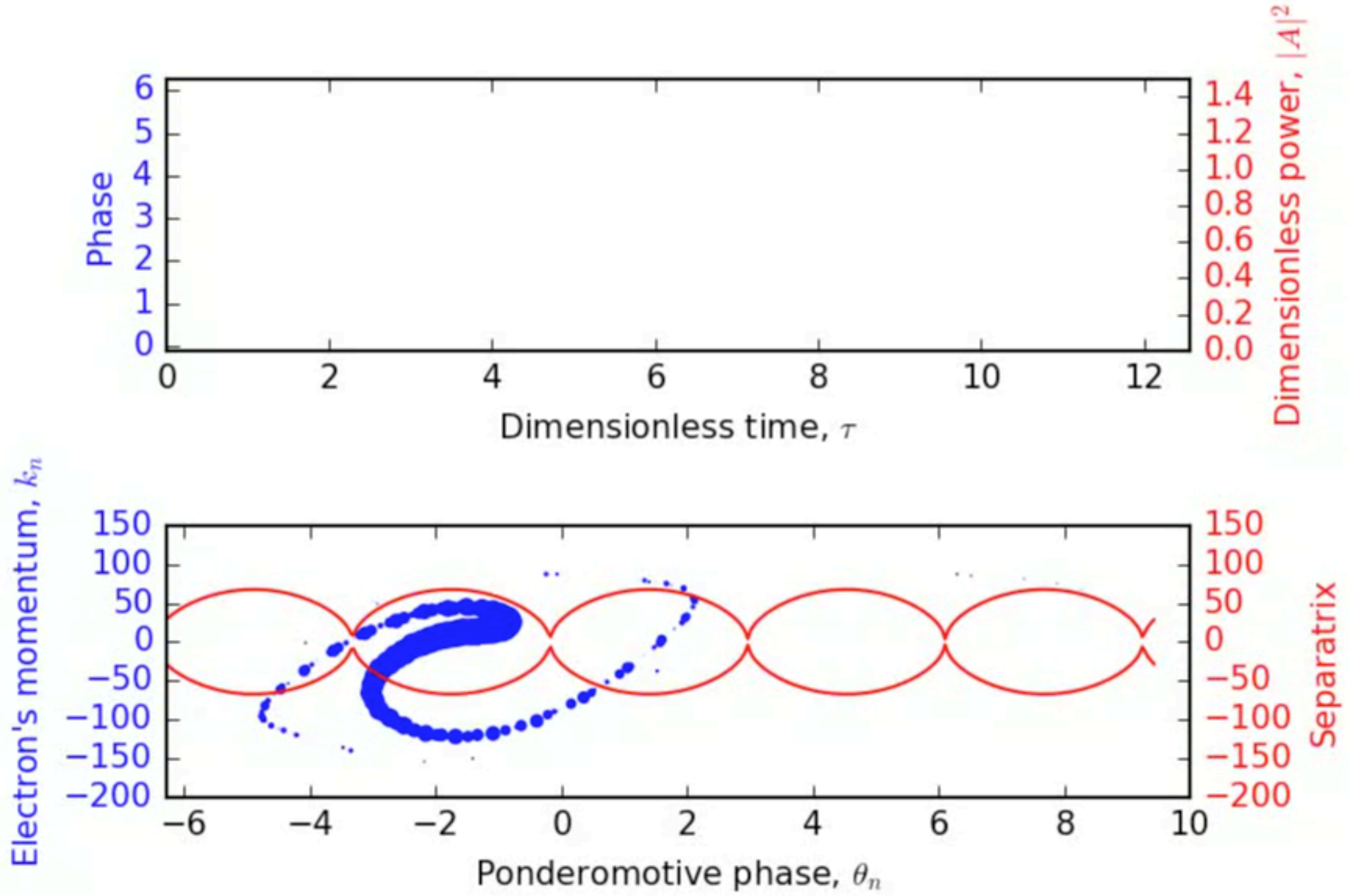


Numerical simulation vs Analytical solution



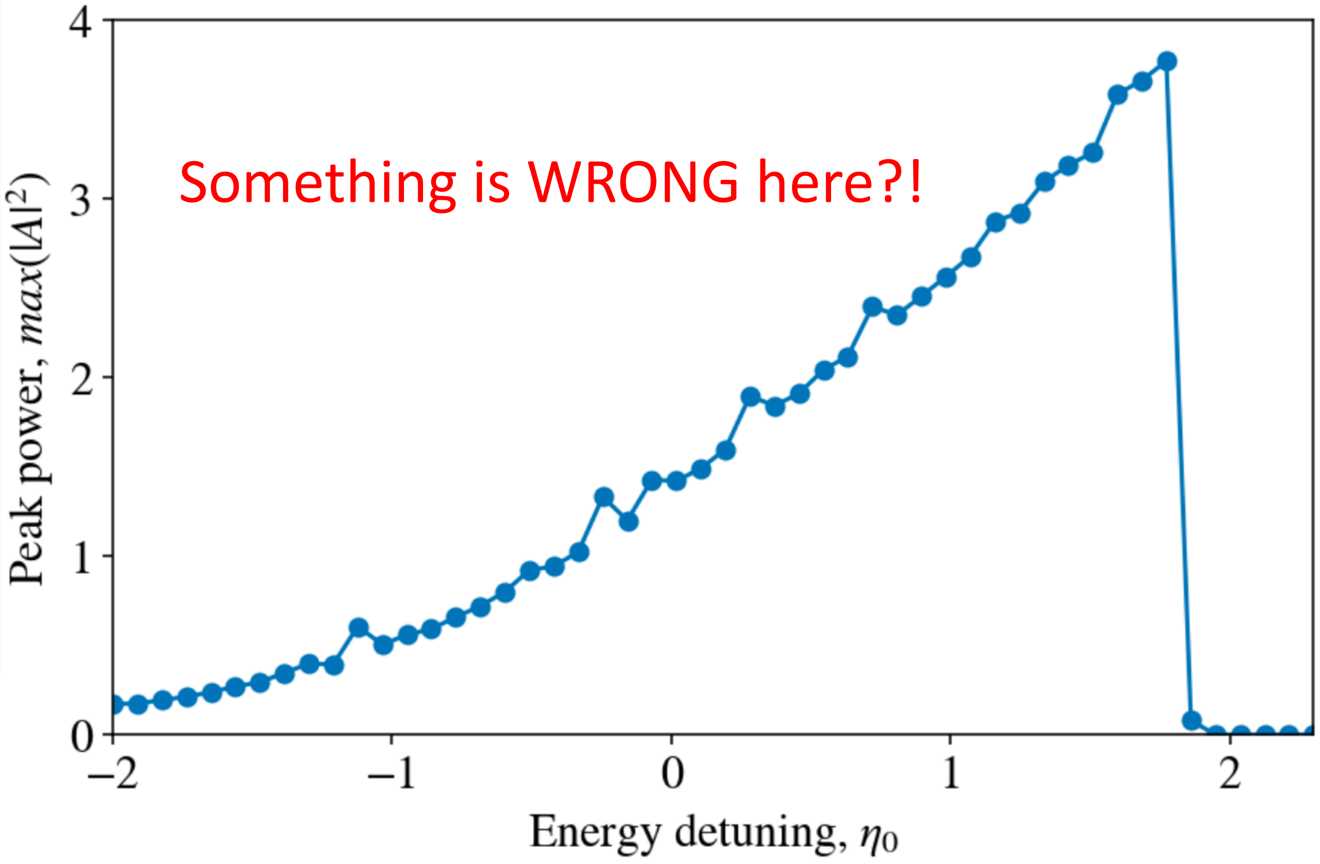
Phase in SASE case (green) vs Seeded case (blue)

Case 7 – A typical animation

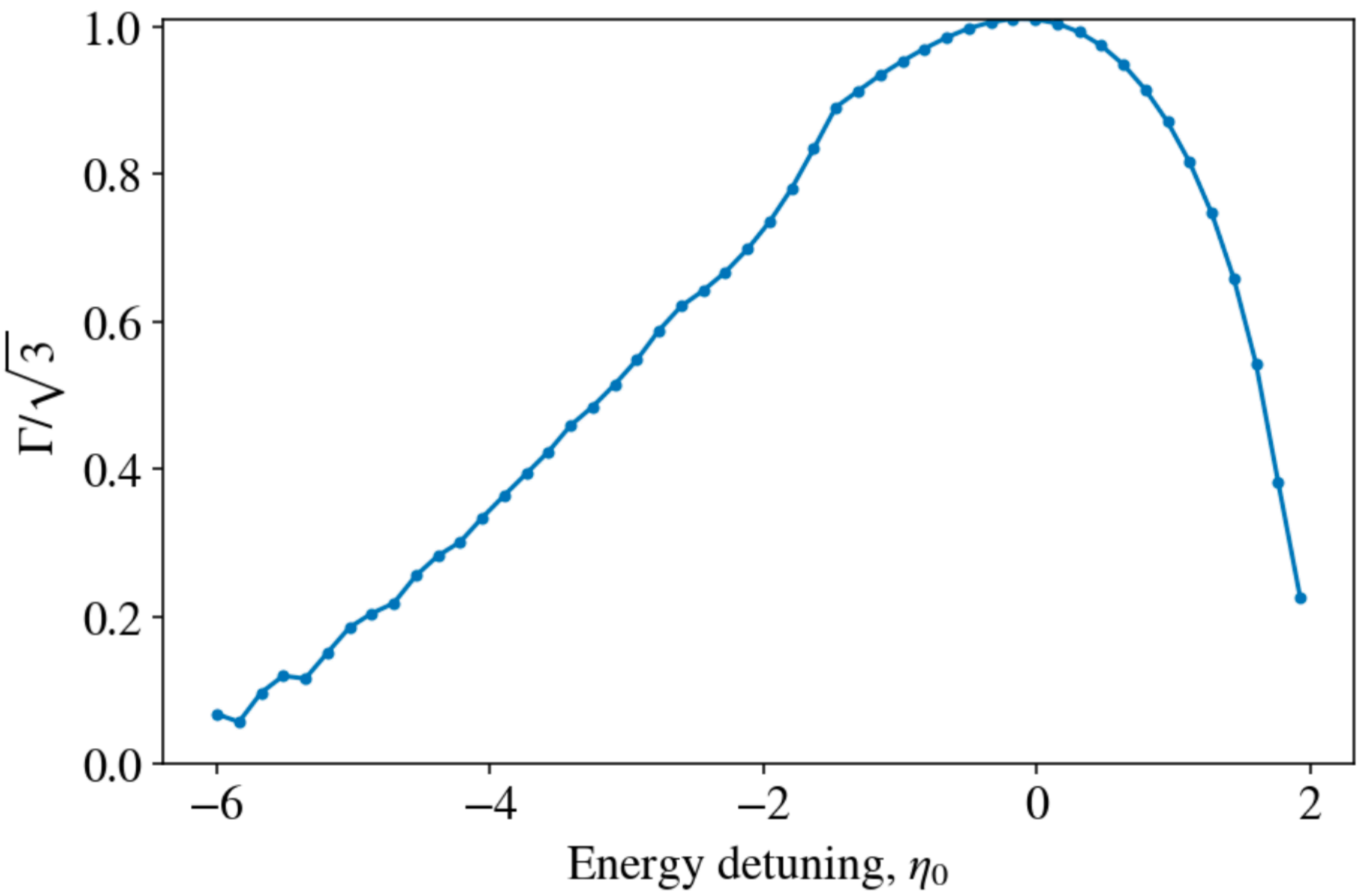


Case 8 – resonance curve

```python
ne = 25
tau = 10*np.pi
A0 = 1.38e-3/2
dtheta = 0
theta0 = np.linspace(-4*np.pi, 4*np.pi, ne, endpoint=False)
theta0 -= dtheta*np.sin(theta0)
res = []
eta = np.linspace(-2,2.3)
for eta0 in eta:
    p0 = eta0*np.ones(ne)
    sol = solve_ivp(rhs, [0, tau],
                    np.concatenate(([A0+0j], theta0, p0)),
                    max_step=0.1)
    res.append(np.max(np.abs(sol.y[0])**2))
plt.plot(eta, res, 'o-')
plt.xlabel(r'Energy detuning, $\eta_0$')
plt.ylabel(r'Peak power, $max(|A|^2)$')
plt.xlim([-2, 2.3])
plt.ylim([0, 4])
plt.show()
```



Case 9 – correct resonance curve (HW)



Observe:

- Maximum gain of $\sqrt{3}$ is reached at zero energy detuning;
- The resonance curve is asymmetric;
- The lower energy case keeps lasing longer!

Numerical simulator – ZFEL code

https://github.com/slaclab/zfel.git

ZFEL package structure

- `sase1d_input_part.sase` performs 1D FEL simulation
  - `.sase.params_calc` performs FEL normalization
  - `.general_load_bucket.general_load_bucket` performs (θ_n, η_n) distribution generation
  - `.sase.FEL_process` performs dimensionless FEL calculations
  - `.sase.final_calc` converts dimensionless results back to physical units

- Output:
  - z, power_z, s, power_s, rho, detune, field, field_s, gainLength, resWavelength, thet_out, eta_out, bunching, spectrum, freq, Ns, history

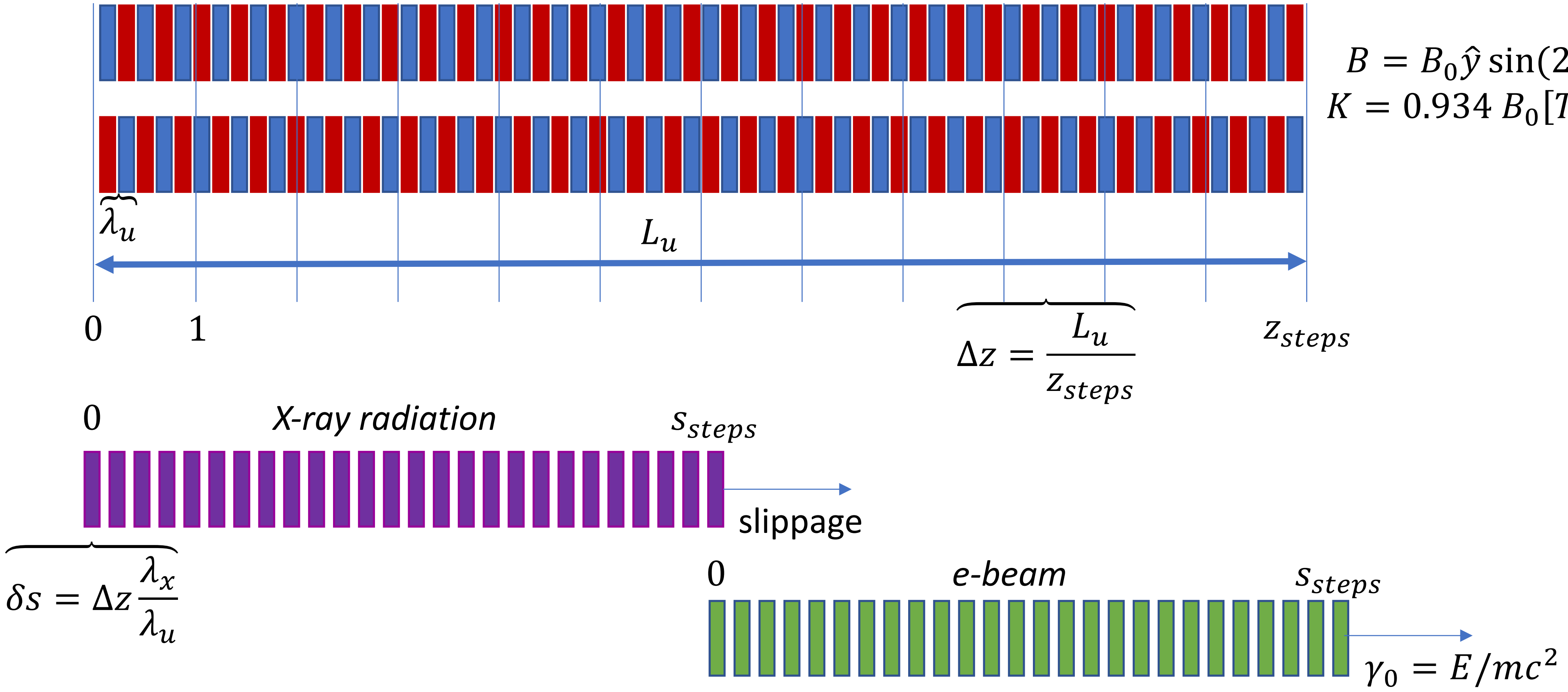
SASE 1D FEL run function: Input

Electron beam description		X-ray radiation description	
energy	electron energy [eV]	s_steps	n-sample points along bunch length
eSpread	relative rms energy spread [ ]	radWavelength	seed wavelength? [meter], used only in single-freuqency runs
emitN	normalized transverse emittance [m-rad]	iopt	'sase' or 'seeded'
currentMax	peak current [Ampere]	P0	small seed input power [W]
beta	mean beta [meter]		

Undulator description		Technical description	
z_steps	n-sample points along undulator	Nruns	not implemented yet
unduPeriod	undulator period [meter]	npart	n-macro-particles per bucket
unduK	undulator parameter, array of K [ ]	constseed	use constant random seed for reproducibility, 1 Yes, 0 No
unduL	length of undulator [meter]	particle_position	particle distro with positions in meter and eta
dEdz	not implemented yet	hist_rule	different rules to select number of intervals to generate the histogram of eta value in a bucket

Graphical representation

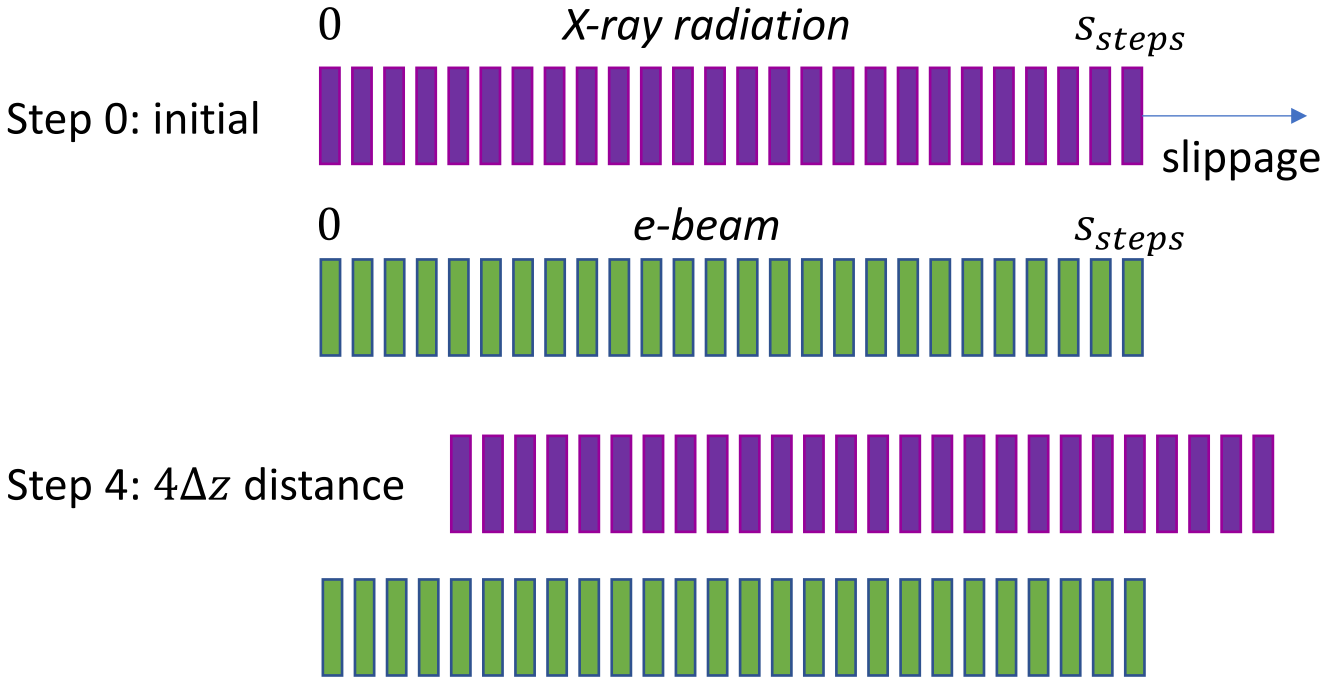
$$B = B_0 \hat{y} \sin(2\pi z/\lambda_u)$$
$$K = 0.934\, B_0[T]\lambda_u[cm]$$

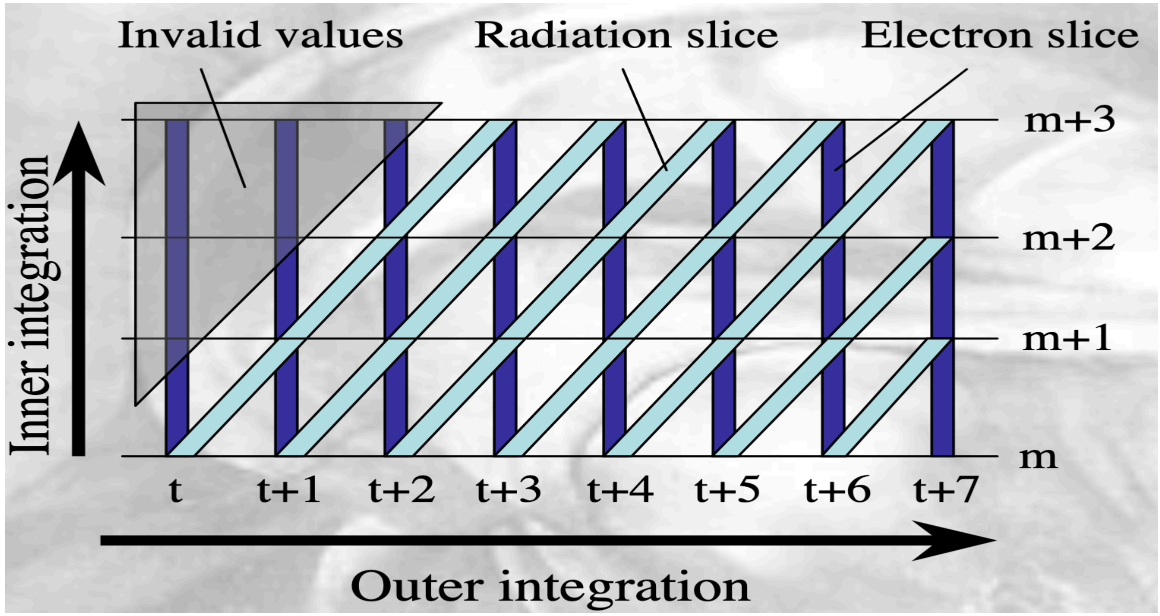


Normalized emittance: $\varepsilon_n = \frac{\varepsilon}{\gamma_0}$; mean beta: $\bar{\beta} = \frac{\sigma^2}{\gamma_0 \varepsilon_n}$

Slippage

- The FEL resonance condition $(k + k_u)z - \omega_x t = const$ means that an x-ray radiation overtakes e-beam by $\delta s = \lambda_x$ every $\Delta z = \lambda_u$;
- Therefore, $\delta s = \Delta z\ \lambda_x / \lambda_u$ is the distance that the radiation overtakes the e-beam every Δz integration step.





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ZFEL code setup

Undulator description

```
unduPeriod = 1.86e-2  # undulator period []
unduL = 3500*unduPeriod  # undulator length [m]
z_steps = 100  # n-sample points along undulator; it also defines time step along bunch
unduK = np.full(z_steps, np.sqrt(2)*0.86)  # undulator parametr
dEdz = None  # not implemented
```

Electron beam description

```
currentMax = 3e3  # maximum current [A]
energy = 12e9  # electron energy [eV]
eSpread = 1.5e-4  # relative rms energy spread sigma_E/E
emitN = 0.2e-6  # normalized emittance [m-rad]
beta = 15  # mean beta function of the beam [m]
```

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ZFEL code setup cont'd

X-ray radiation description

```
iopt = 'sase'  # The only mode of operation
P0 = 0e4  # small seed input power [W]
radWavelength =  2.934e-11 # seed wavelength? [meter],
                           # used only in single-freuqency runs
s_steps = 10000 # n-samples along bunch; tail is at 0; it also defines bunch length
```

Technical description

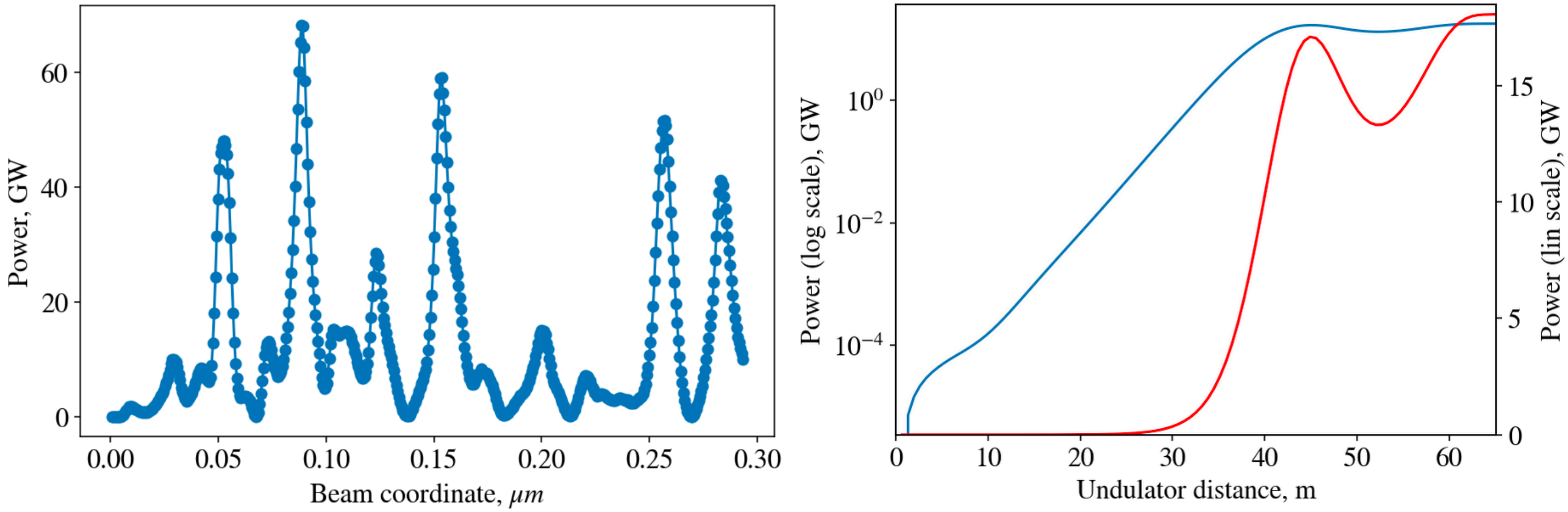
```
Nruns = None  # not implemented
npart = 2**9  # n-macro-particles per bucket
constseed = 1  # use constant random seed for reproducibility,
               # 1 Yes, 0 No
particle_position = None  # particle distro with positions in
                          # meter and eta
hist_rule  = 'square-root'  # 'square-root' or 'sturges' or
                            # 'rice-rule' or 'self-design',
                            # number of intervals to generate
                            # the histogram of eta value in a bucket
```

Execution

Execute

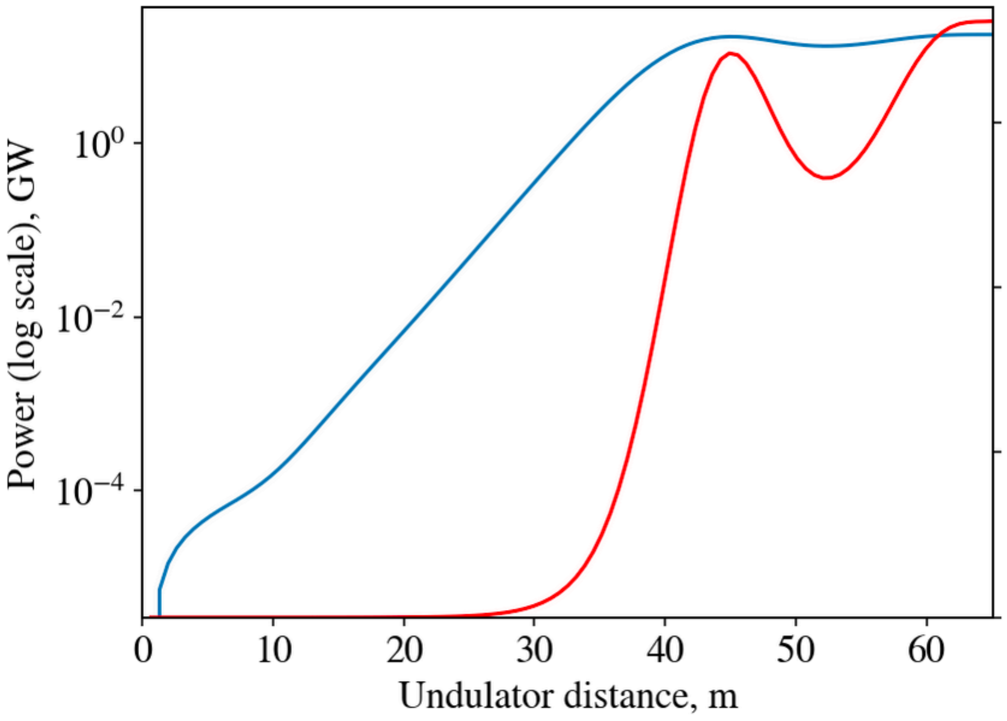
```
# Load input parameters into zfel
inp_struct={'Nruns':Nruns, 'npart':npart,'s_steps':s_steps,'z_steps':z_steps,\
            'energy':energy,'eSpread':eSpread, 'dEdz':dEdz,\
            'emitN':emitN,'currentMax':currentMax,'beta':beta,\
            'unduPeriod':unduPeriod,'unduK':unduK,'unduL':unduL, \
            'radWavelength':radWavelength,'iopt':iopt,'P0':P0,\
            'constseed':constseed,'particle_position':particle_position,'hist_rule':hist_rule}
z,power_z,s,power_s,rho,detune,field,\
field_s,gainLength,resWavelength,\
thet_out,eta_out,bunching,spectrum,freq,Ns,history = sase1d_input_part.sase(inp_struct)
```

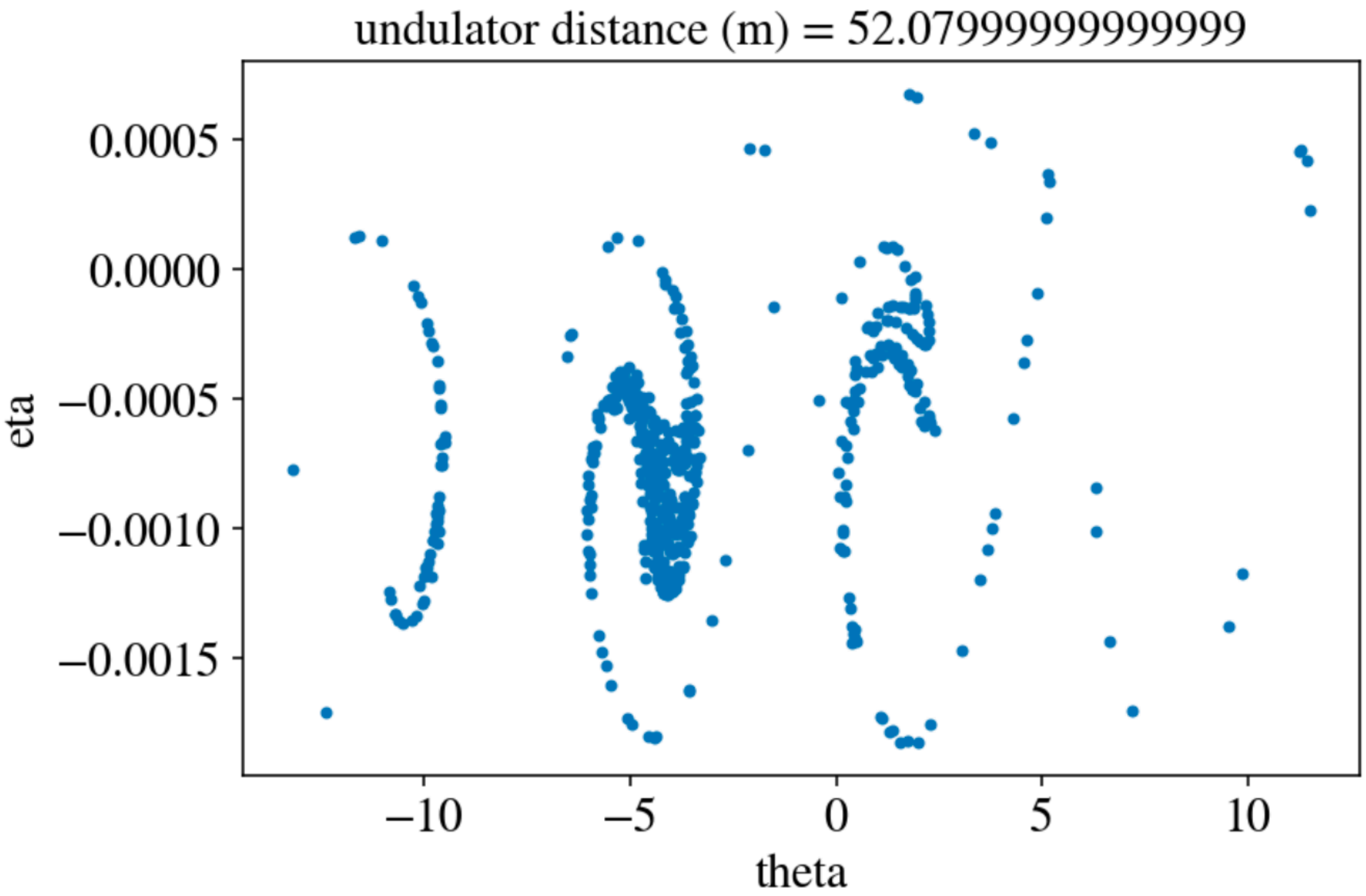
Results: x-ray power



Simulated power at a MaRIE-like x-ray FEL in the 1D approximation. Strong SASE power fluctuations are present. Averaged power exhibits expected characteristics of initial lethargy, exponential growth and saturation.

Results: phase space distribution





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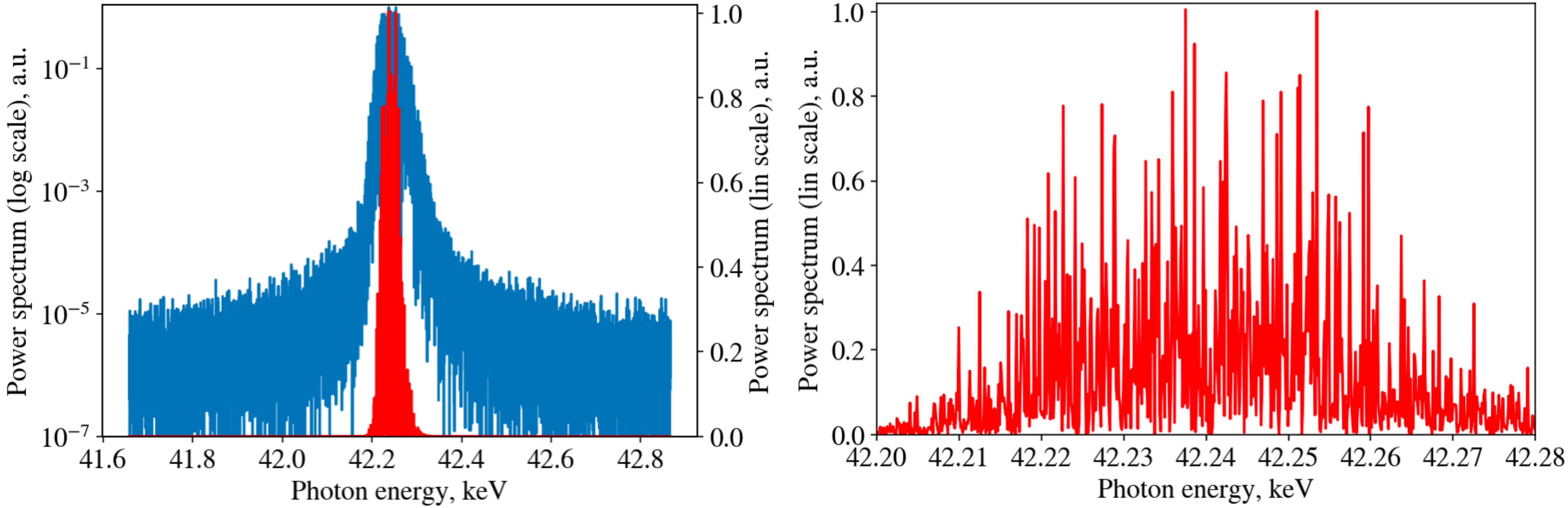
Results: spectral analysis



Simulated power spectrum at a MaRIE-like x-ray FEL in the 1D approximation. Strong SASE power fluctuations corresponding to the spectrum with a relative bandwidth of $4{\times}10^{-4}$. The contrast of the resonant radiation vs incoherent background is about 10^{-4}.

Results: Transverse dynamics?